Practical: 1

Make a geometric plot to show that the nth roots of unity are equally spaced points that lie on the unit circle c(0,1)={z:|z|=1} and form vertices of a regular polygon with n sides.

```
kill(all);
```

done

```
root:solve(z^3=1,z);
```

$$\left[z=\frac{\sqrt{3}\ \%i-1}{2},z=-\left(\frac{\sqrt{3}\ \%i+1}{2}\right),z=1\right]$$

```
sol:map(rhs,root);
```

$$\left[\frac{\sqrt{3}\ \%i-1}{2},-\left(\frac{\sqrt{3}\ \%i+1}{2}\right),1\right]$$

```
rsol:map(realpart,sol);
isol:map(imagpart,sol);
```

$$\left[-\left(\frac{1}{2}\right),-\left(\frac{1}{2}\right),1\right] \qquad \left[\frac{\sqrt{3}}{2},-\left(\frac{\sqrt{3}}{2}\right),0\right]$$

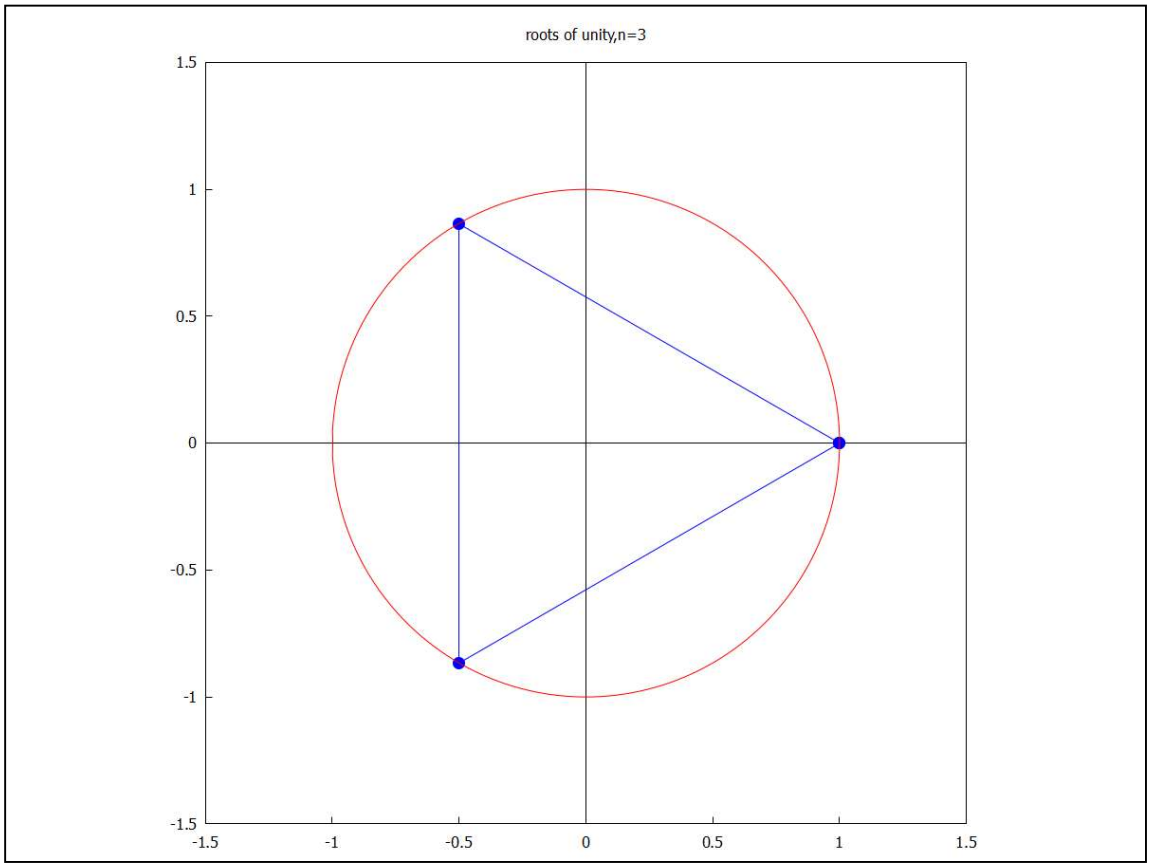
```
rsol1:cons(1,rsol);
```

$$\left[1,-\left(\frac{1}{2}\right),-\left(\frac{1}{2}\right),1\right]$$

```
isol1:cons(0,isol);
```

$$\left[0,\frac{\sqrt{3}}{2},-\left(\frac{\sqrt{3}}{2}\right),0\right]$$

```
wxdraw2d(
title=concat("roots of unity,n=",3 ),
xaxis=true,xaxis_type=solid,xrange=[-1.5,1.5],
yaxis=true,yaxis_type=solid,yrange=[-1.5,1.5],
proportional_axes=xy,
point_size=2,
point_type=7,
   points_joined=true,
points(rsol1, isol1),
color=red,
nticks=200,
parametric(cos(t),sin(t),t,0,2·%pi));
```



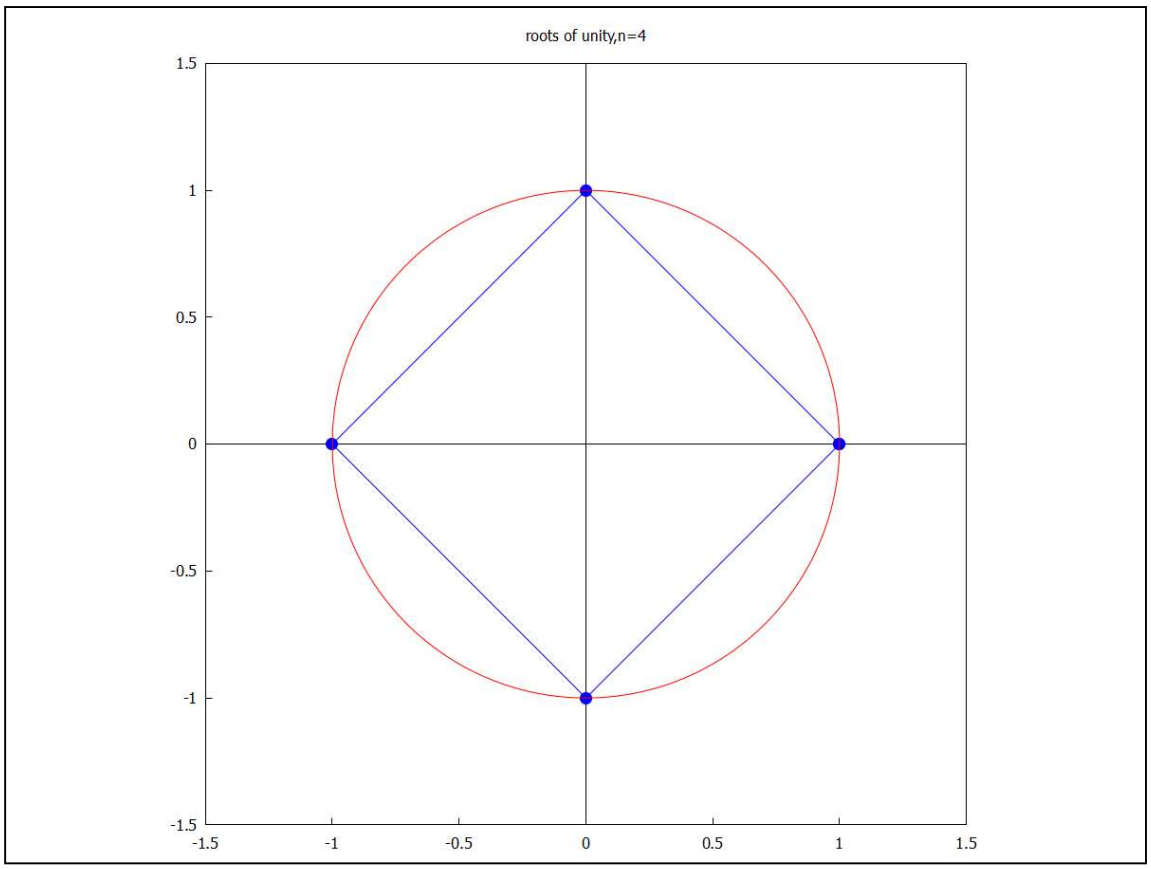

```
kill(all);
```

done

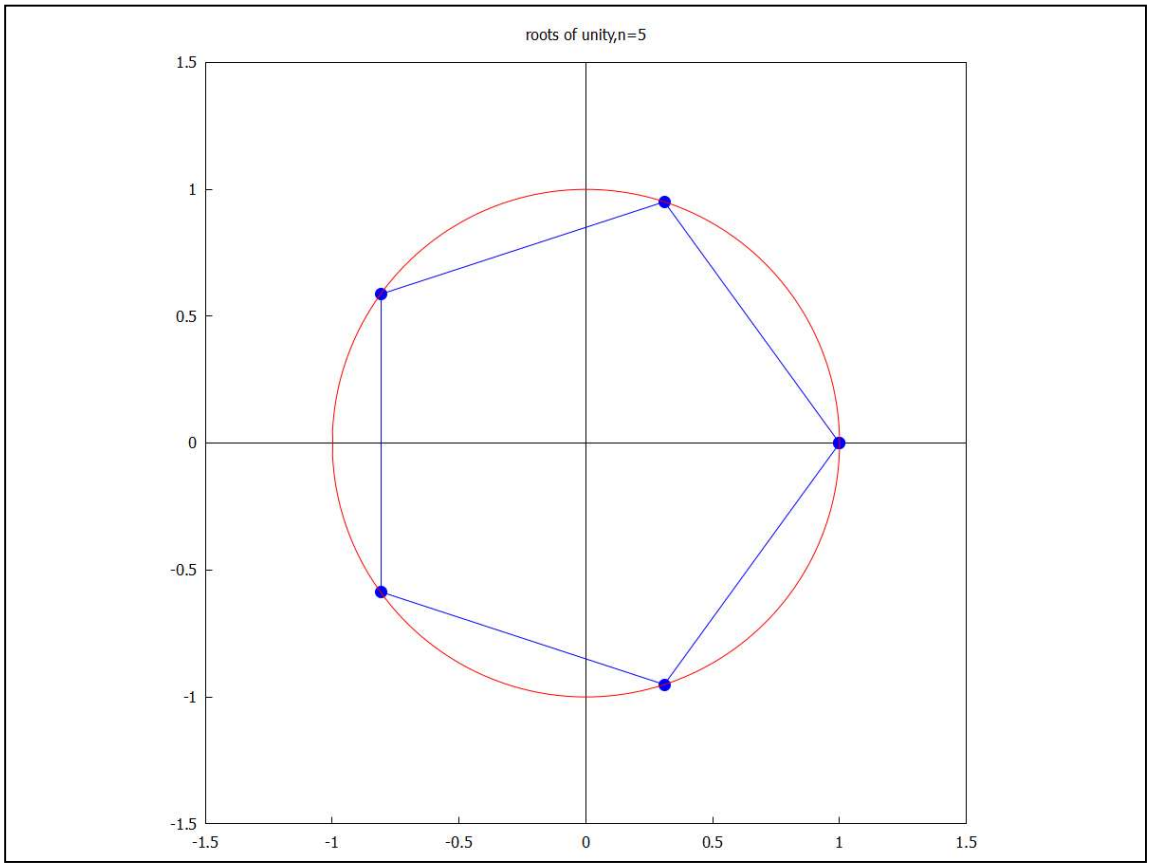
```
plotRoots(n):=block(
root:solve(z^n=1,z),
sol:map(rhs,root),
rsol:map(realpart,sol),
isol:map(imagpart,sol),
rsol1:cons(1,rsol),
isol1:cons(0,isol),
wxdraw2d(
title=concat("roots of unity,n=",n ),
xaxis=true,xaxis_type=solid,xrange=[-1.5,1.5],
yaxis=true,yaxis_type=solid,yrange=[-1.5,1.5],
proportional_axes=xy,
point_size=2,
point_type=7,
points_joined=true,
points(rsol1, isol1),
color=red,
nticks=200,
parametric(cos(t),sin(t),t,0,2·%pi)
    ));
```

$$\text{plotRoots}(n):=\text{block}(root:\text{solve}(z^{n}=1,z),sol:\text{map}(rhs,root),$$

```
rsol:map(realpart,sol),isol:map(imagpart,sol),rsol1:cons(1,rsol),
isol1:cons(0,isol),wxdraw2d(title=concat(roots of unity,n=,n),xaxis=
true,xaxis_type=solid,xrange=[-1.5,1.5],yaxis=true,yaxis_type=
solid,yrange=[-1.5,1.5],proportional_axes=xy,point_size=2,
point_type=7,points_joined=true,points(rsol1,isol1),color=red,nticks
=200,parametric(cos(t),sin(t),t,0,2 π)))
plotRoots(4);
```




```
plotRoots(5);
```



Practical: 2

Find all the solutions of the equation z^3=8i and represent these geometrically.

```
kill(all);
```

done

```
roots:solve(z^3=8·%i, z);
```

$$[z=(-1)^{1/6}\sqrt{3}\,\%i-(-1)^{1/6},\ z=-\left((-1)^{1/6}\sqrt{3}\,\%i\right)-(-1)^{1/6},\ z=2\,(-1)^{1/6}]$$

```
sol:map(rhs,roots);
```

$$[(-1)^{1/6}\sqrt{3}\,\%i-(-1)^{1/6},\ -\left((-1)^{1/6}\sqrt{3}\,\%i\right)-(-1)^{1/6},\ 2\,(-1)^{1/6}]$$

```
rsol:map(realpart,sol);
```

$$[-\sqrt{3},0,\sqrt{3}]$$

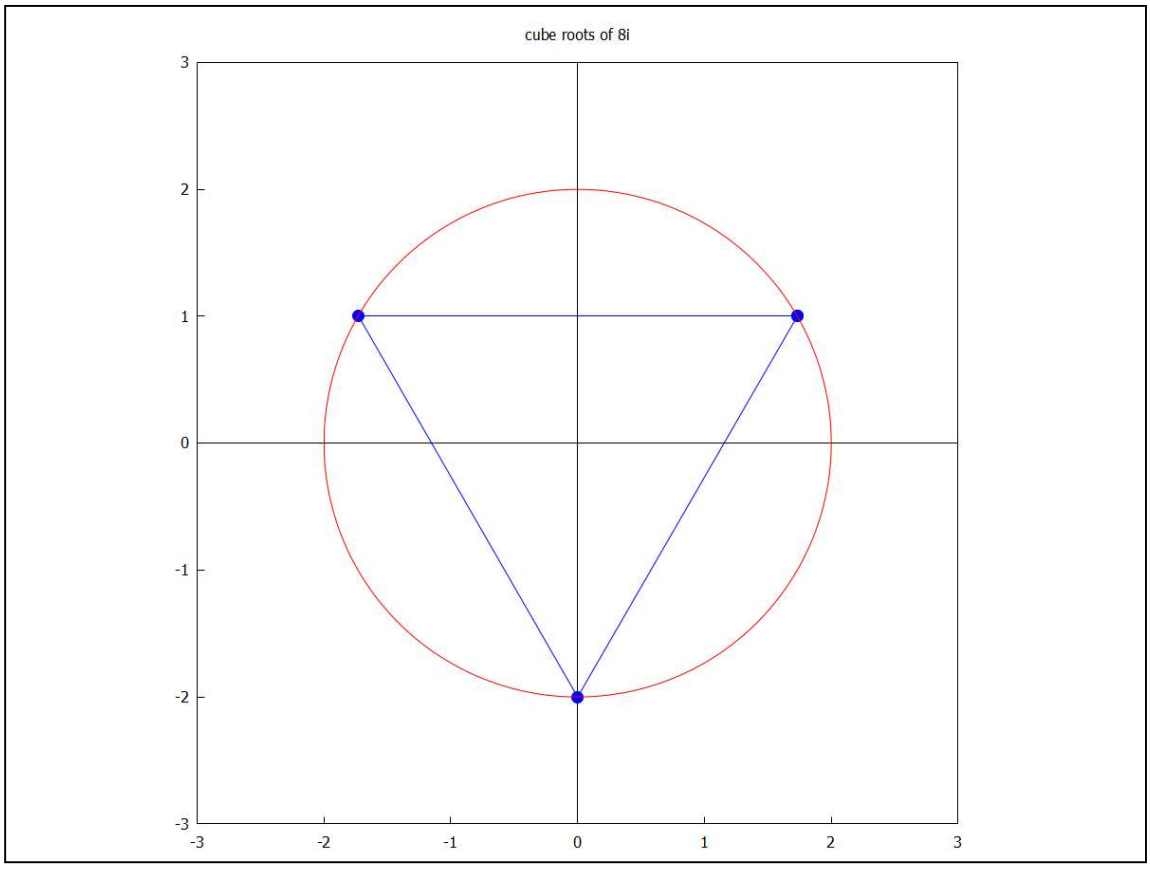
```
isol:map(imagpart,sol);
```

$$[1,-2,1]$$

```
rsol1:cons(rsol[3],rsol);
isol1:cons(isol[3],isol);
```

$$[\sqrt{3},-\sqrt{3},0,\sqrt{3}] \qquad [1,1,-2,1]$$

```
wxdraw2d(
title = concat("cube roots of 8i"),
xaxis = true, xaxis_type = solid, xrange = [-3, 3],
yaxis = true, yaxis_type = solid, yrange = [-3, 3],
proportional_axes = xy,
color = magenta,
head_length = 0.3,
head_angle = 30,
color = blue,
point_size = 2,
point_type = 7,
points_joined = true,
points(rsol1, isol1),
color = red,
nticks = 200,
parametric(2·cos(t), 2·sin(t), t, 0, 2·%pi));
```



Practical: 3

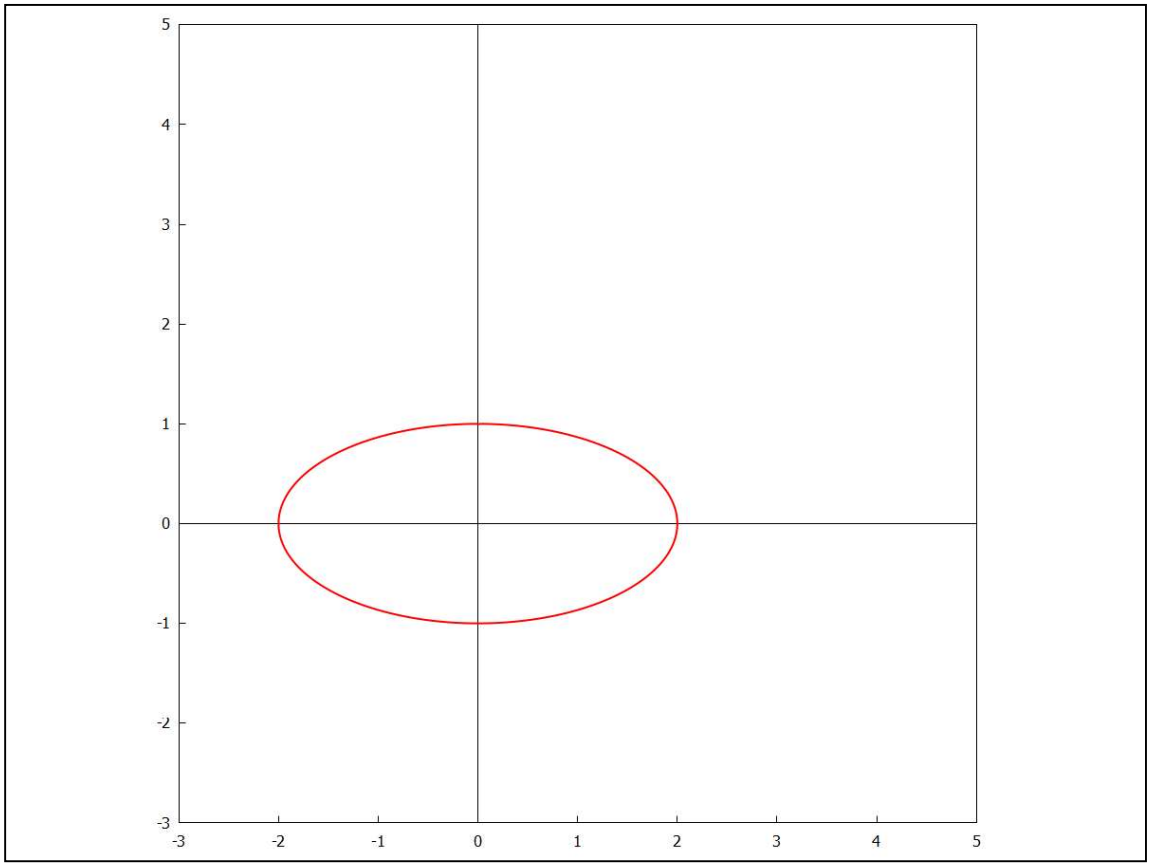
```
kill(all);
```

done

```
f(t):= 2·cos(t)+%i·sin(t);
```

$$f(t):=2\cos(t)+\%i\sin(t)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-3,5],
yaxis=true,yaxis_type=solid,yrange=[-3,5],
proportional_axes=xy,
line_width=2,
nticks=600,
color=red,
parametric(realpart(f(t)),imagpart(f(t)),t,0,2·%pi));
```



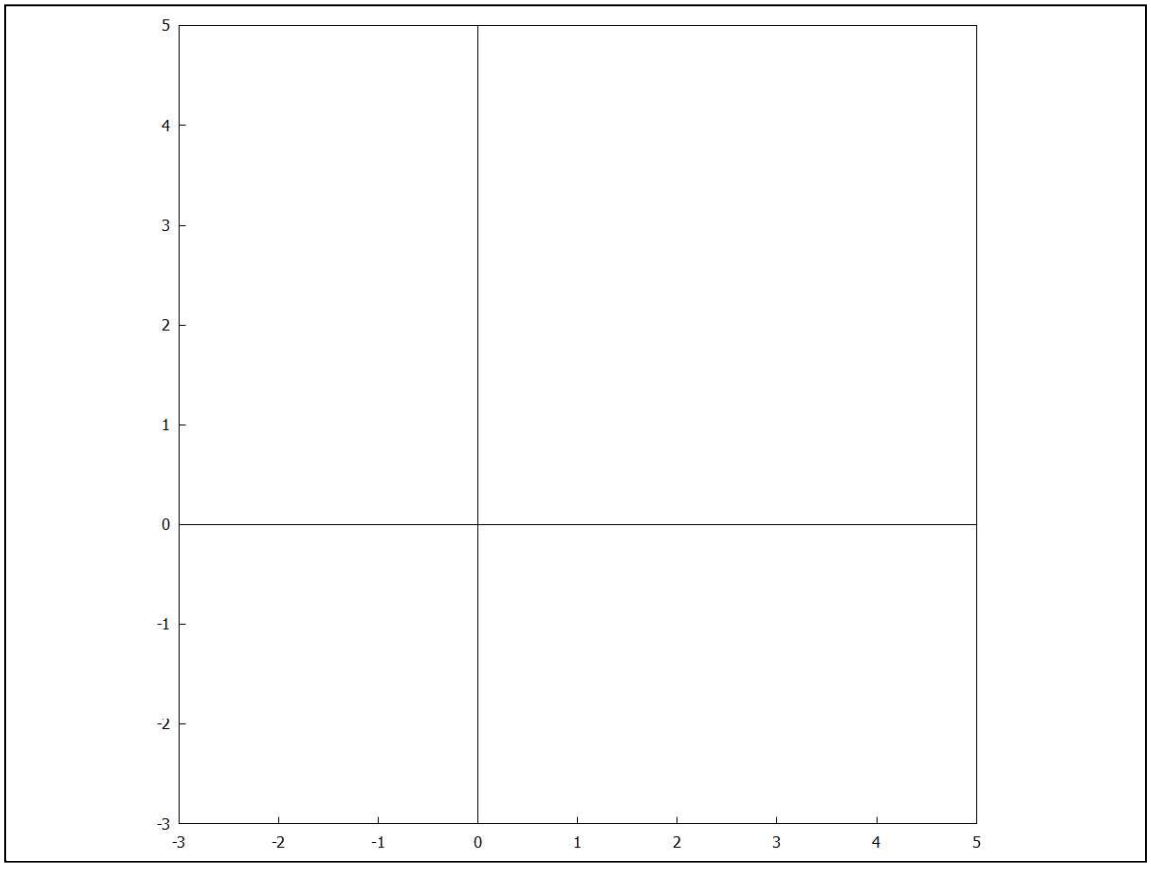

```
theta:-%pi/6;
```

$$-\left(\frac{\pi}{6}\right)$$

```
r(t):= s(t)·exp(%i·-%pi/6);
```

$$r(t) := s(t)\exp\left(\frac{\%i\,(-\pi)}{6}\right)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-3,5],
yaxis=true,yaxis_type=solid,yrange=[-3,5],
proportional_axes=xy,
line_width=2,
nticks=200,
parametric(realpart(r(t)),imagpart(r(t)),t,0,2·%pi));
```



Practical:4

```
kill(all);
```

done

```
f(z):=block(
[x,y],
x:realpart(z),
y:imagpart(z),
w:rectform((3-4·%i)·(x+%i·y)+(6+2·%i)));
```

f(z):=block([x,y],x:realpart(z),y:imagpart(z),w:
rectform((3−4 %i)(x+%i y)+(6+2 %i)))

```
c:f(-1-%i);
```

3 %i−1

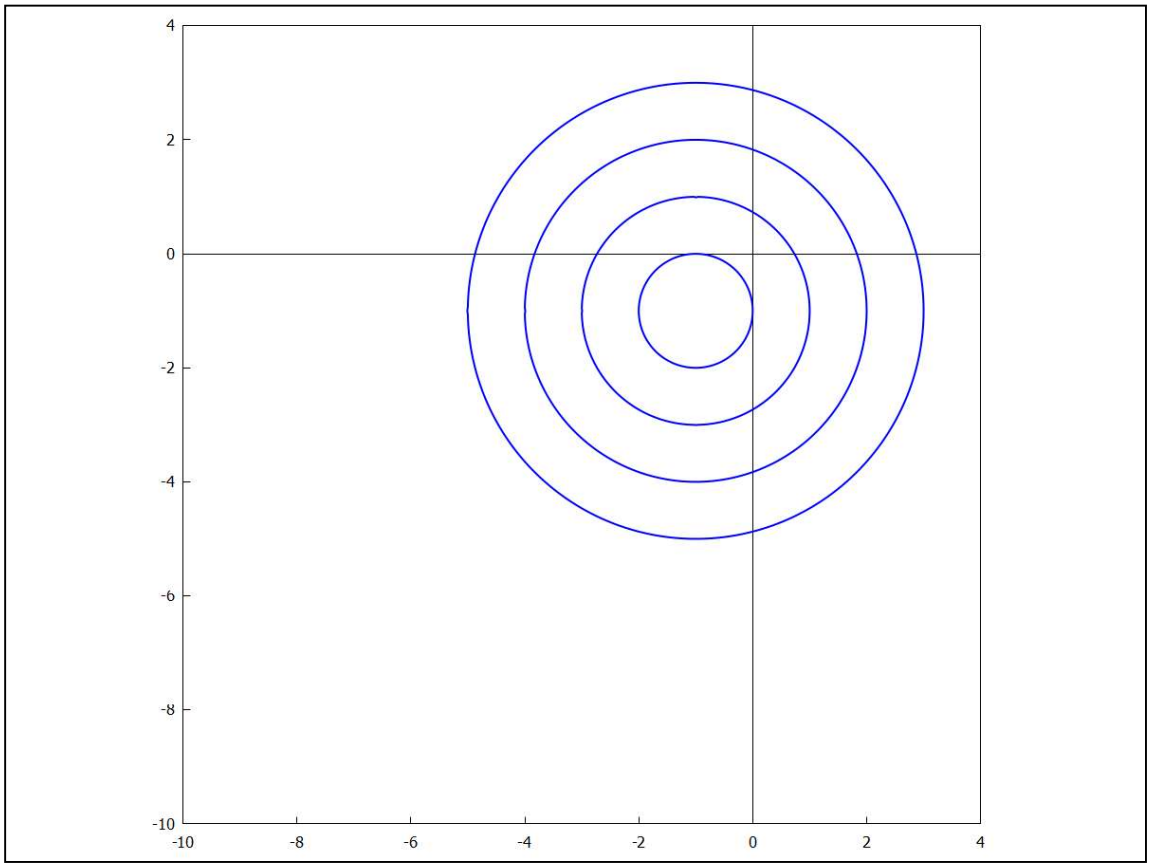
```
r(t,s):=-1-%i+s·(cos(t)+%i·sin(t));
```

r(t,s):=−1−%i+s (cos(t)+%i sin(t))

```
zdomain:makelist(parametric(realpart(r(t,s)),imagpart(r(t,s)),t,0,2·%pi),s,0,4);
```

[parametric(−1,−1,t,0,2 π),
parametric(cos(t)−1,sin(t)−1,t,0,2 π),
parametric(2 cos(t)−1,2 sin(t)−1,t,0,2 π),
parametric(3 cos(t)−1,3 sin(t)−1,t,0,2 π),
parametric(4 cos(t)−1,4 sin(t)−1,t,0,2 π)]

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-10,4],
yaxis=true,yaxis_type=solid,yrange=[-10,4],
proportional_axes=xy,
line_width=2,
nticks=600,
zdomain);
```



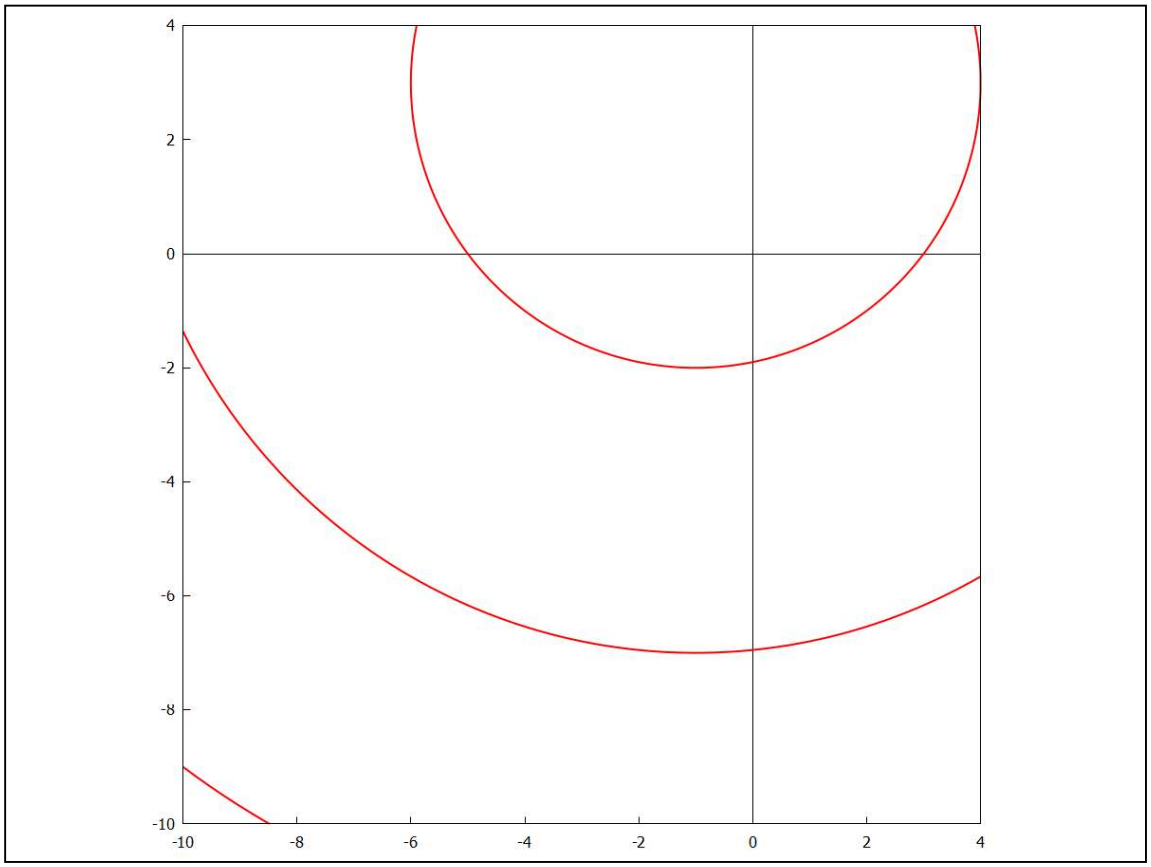

```
w(t,s):=f(r(t,s));
```

$$w(t,s):=f(r(t,s))$$

```
wdomain:makelist(parametric(realpart(w(t,s)),imagpart(w(t,s)),t,0,2·%pi),s,0,4);
```

$$[\text{parametric}(-1,3,t,0,2\,\pi),\text{parametric}(4\,(\sin(t)-1)+3\,(\cos(t)-1)+6,3\,(\sin(t)-1)-4\,(\cos(t)-1)+2,t,0,2\,\pi),\text{parametric}(4\,(2\sin(t)-1)+3\,(2\cos(t)-1)+6,3\,(2\sin(t)-1)-4\,(2\cos(t)-1)+2,t,0,2\,\pi),\text{parametric}(4\,(3\sin(t)-1)+3\,(3\cos(t)-1)+6,3\,(3\sin(t)-1)-4\,(3\cos(t)-1)+2,t,0,2\,\pi),\text{parametric}(4\,(4\sin(t)-1)+3\,(4\cos(t)-1)+6,3\,(4\sin(t)-1)-4\,(4\cos(t)-1)+2,t,0,2\,\pi)]$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-10,4],
yaxis=true,yaxis_type=solid,yrange=[-10,4],
proportional_axes=xy,
line_width=2,
nticks=600,
   color=red,
wdomain);
```



Practical : 5

```
kill(all);
```

done

```
f(z):=block([x,y],
x:realpart(z),
y:imagpart(z),
w:rectform((-1+%i)·(x+%i·y)+(-2+3·%i)));
```

f(z):=block([x,y],x:realpart(z),y:imagpart(z),w:rectform((-1+%i)(x+%i y)+(-2+3 %i)))

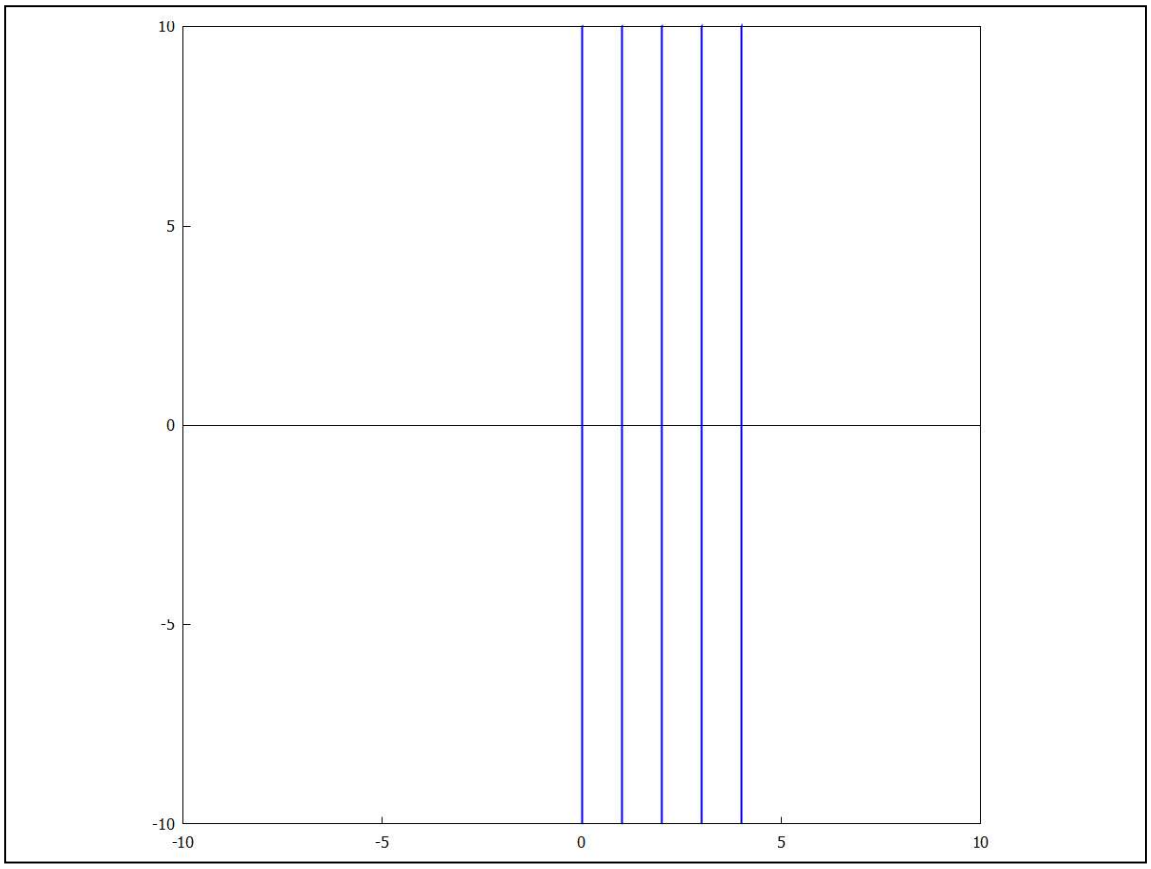
```
r(t,s):=(s+%i·t);
```

r(t,s):=s+%i t

```
zdomain:makelist(parametric(realpart(r(t,s)),imagpart(r(t,s)),t,-10,10),s,0,4);
```

[parametric(0,t,t,-10,10),parametric(1,t,t,-10,10),parametric(2,t,t,-10,10),parametric(3,t,t,-10,10),parametric(4,t,t,-10,10)]

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-10,10],
yaxis=true,yaxis_type=solid,yrange=[-10,10],
proportional_axes=xy,
line_width=2,
nticks=600,
zdomain);
```



Practical : 6

```
kill(all);
```

done

```
f(z):=block(
[x,y],
x:realpart(z),
y:imagpart(z),
w:rectform(1/(x+%i·y)));
```

$$f(z):=\text{block}\left(\left[x,y\right],x:\text{realpart}\left(z\right),y:\text{imagpart}\left(z\right),w:\text{rectform}\left(\frac{1}{x+\%\text{i}\,y}\right)\right)$$

```
f(1);
```

(%o1) 1

```
f(%i);
```

(%o2) −%i

```
f(1+%i);
```

(%o3) $\frac{1}{2} - \frac{\%i}{2}$

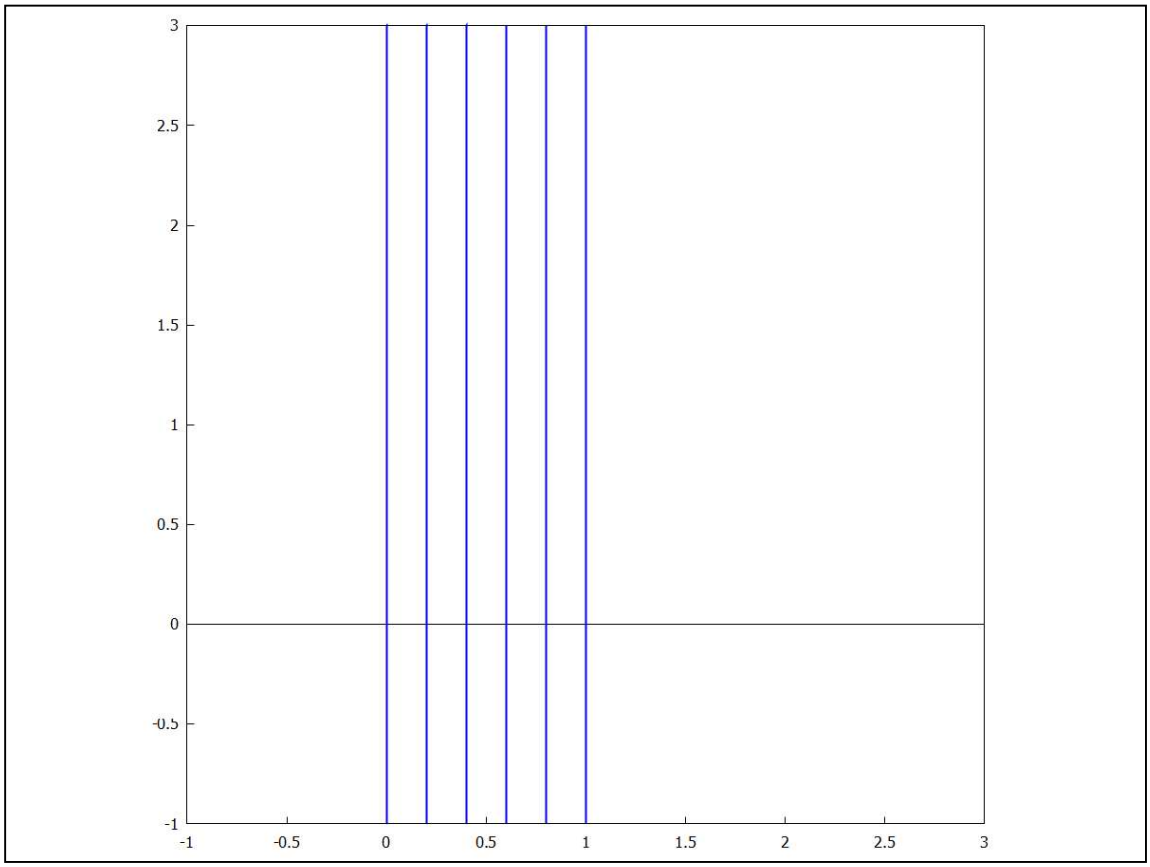
```
z(t,s):=(s+%i·t);
```

(%o4) $z(t,s):=s+\%i\ t$

```
zdomain:makelist(parametric(realpart(z(t,s)),imagpart(z(t,s)),t,-1,3),s,0,1,1/5);
```

(%o5) $[\text{parametric}(0,t,t,-1,3),\text{parametric}\left(\frac{1}{5},t,t,-1,3\right),$
$\text{parametric}\left(\frac{2}{5},t,t,-1,3\right),\text{parametric}\left(\frac{3}{5},t,t,-1,3\right),$
$\text{parametric}\left(\frac{4}{5},t,t,-1,3\right),\text{parametric}(1,t,t,-1,3)]$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-1,3],
yaxis=true,yaxis_type=solid,yrange=[-1,3],
proportional_axes=xy,
line_width=2,
nticks=600,
zdomain);
```



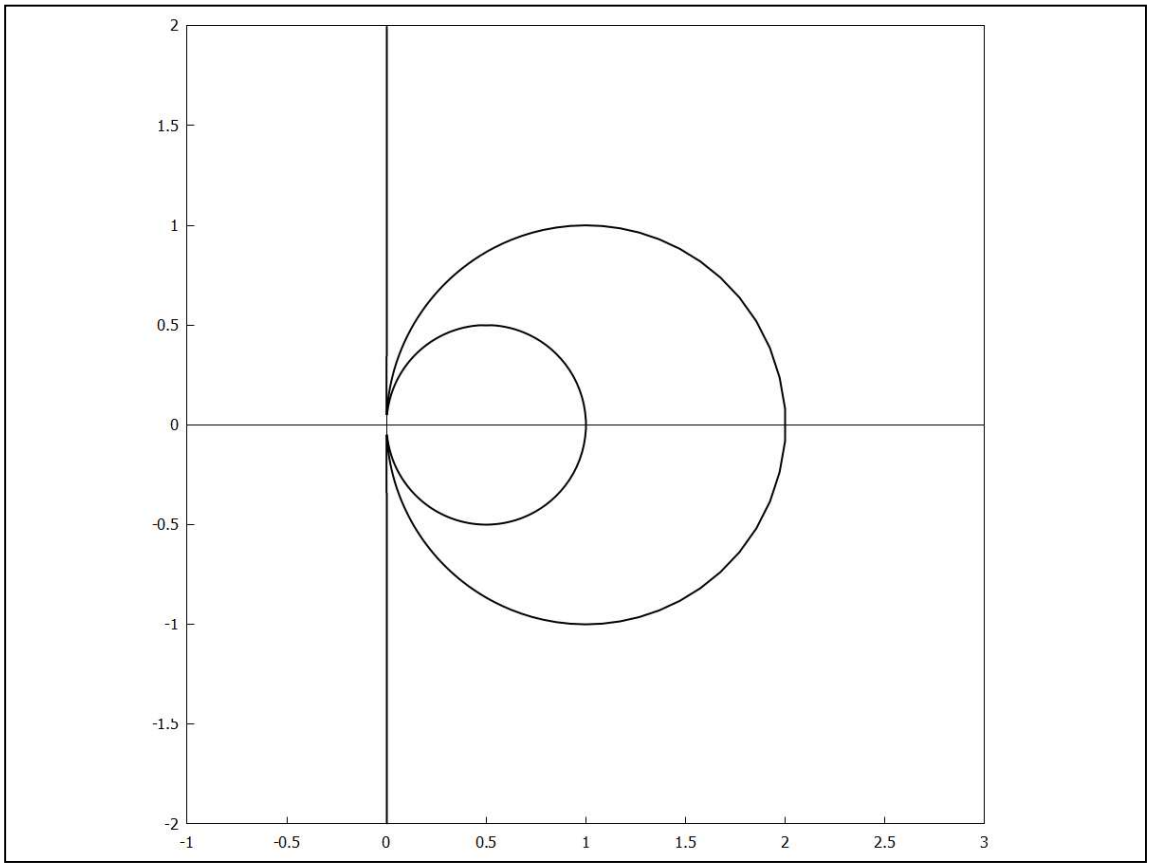

```
w(t,s):=f(z(t,s));
```

$$w(t,s):=f(z(t,s))$$

```
wdomain:makelist(parametric(realpart(w(t,s)),imagpart(w(t,s)),t,-20,20),s,0,1,1/2);
```

$$\left[\text{parametric}\left(0,-\left(\frac{1}{t}\right),t,-20,20\right),\ \text{parametric}\left(\frac{1}{2\left(t^2+\frac{1}{4}\right)},-\left(\frac{t}{t^2+\frac{1}{4}}\right),t,-20,20\right),\ \text{parametric}\left(\frac{1}{t^2+1},-\left(\frac{t}{t^2+1}\right),t,-20,20\right)\right]$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-1,3],
yaxis=true,yaxis_type=solid,yrange=[-2,2],
proportional_axes=xy,
line_width=2,
color=black,
nticks=1000,
wdomain);
```



Practical: 7

```
kill(all);
```

done

```
f(z):=block(
[x,y],
x:realpart(z),
y:imagpart(z),
w:rectform(1/(x+%i·y)));
```

$$\mathrm{f}(z):=\mathrm{block}\left([x,y],x:\mathrm{realpart}(z),y:\mathrm{imagpart}(z),w:\mathrm{rectform}\left(\frac{1}{x+\%\mathrm{i}\ y}\right)\right)$$

```
f(1);
```

1

```
f(2);
```

$$\frac{1}{2}$$

```
z(t,s):=(t+%i·s);
```

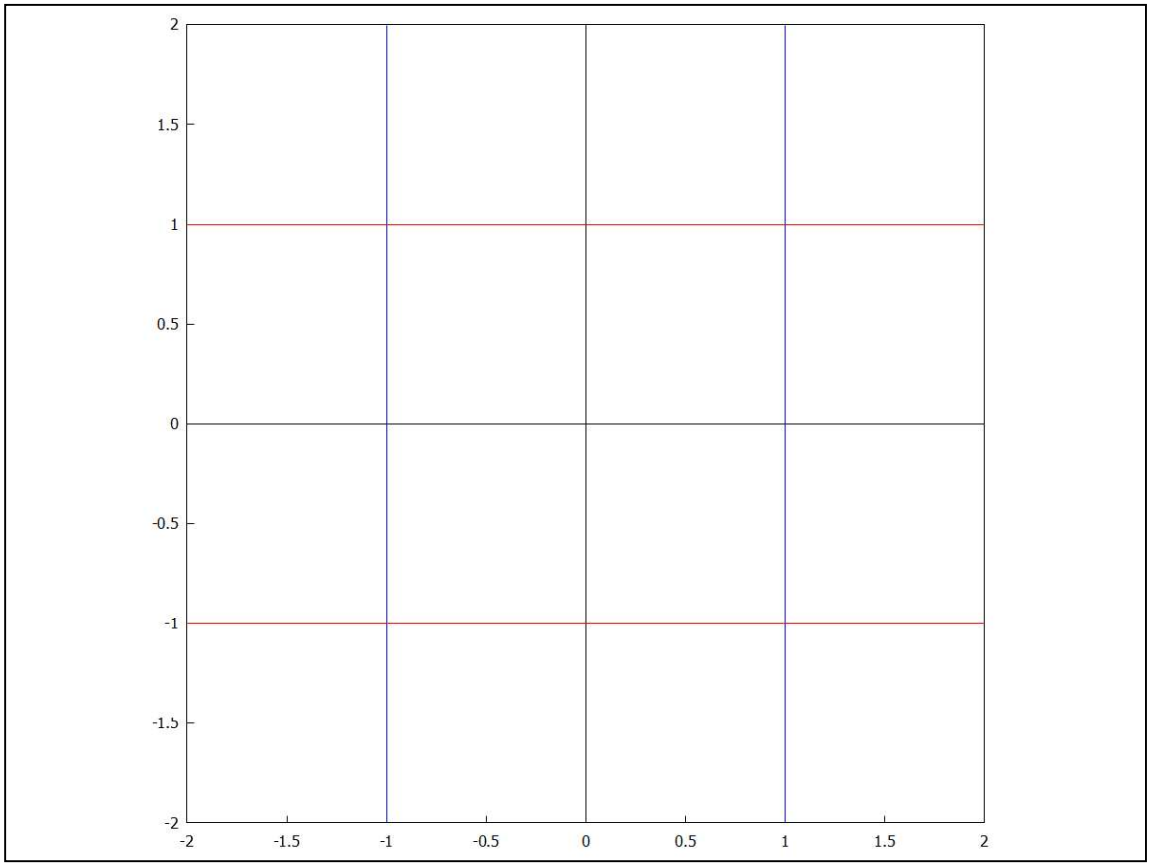
$$\mathrm{z}(t,s):=t+\%\mathrm{i}\ s$$

```
h:makelist(parametric(realpart(z(t,s)),imagpart(z(t,s)),t,-3,3),s,[-1,1]);
v:makelist(parametric(realpart(z(t,s)),imagpart(z(t,s)),s,-3,3),t,[-1,1]);
```

$$\left[\text{parametric}(t,-1,t,-3,3),\text{parametric}(t,1,t,-3,3)\right]$$

$$\left[\text{parametric}(-1,s,s,-3,3),\text{parametric}(1,s,s,-3,3)\right]$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-2,2],
yaxis=true,yaxis_type=solid,yrange=[-2,2],
proportional_axes=xy,
   nticks=600,
v,
color=red,
h);
```




```
w(t,s):=f(z(t,s));
```

$$w(t,s):=f(z(t,s))$$

```
h1:makelist(parametric(realpart(w(t,s)),imagpart(w(t,s)),t,-10,10),s,[-1,1]);
v1:makelist(parametric(realpart(w(t,s)),imagpart(w(t,s)),s,-10,10),t,[-1,1]);
```

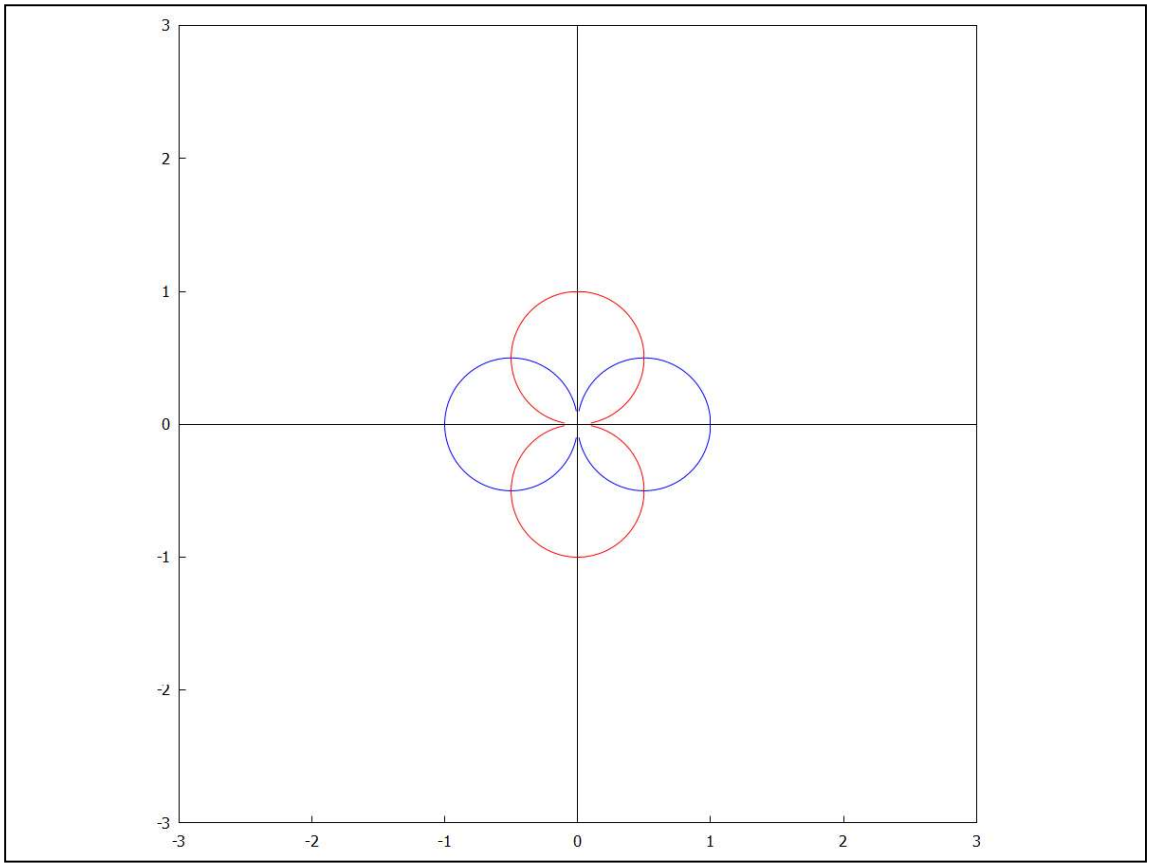
$$\left[\text{parametric}\left(\frac{t}{t^2+1},\frac{1}{t^2+1},t,-10,10\right),\right.$$

$$\text{parametric}\left(\frac{t}{t^2+1},-\left(\frac{1}{t^2+1}\right),t,-10,10\right]\qquad[$$

$$\text{parametric}\left(-\left(\frac{1}{s^2+1}\right),-\left(\frac{s}{s^2+1}\right),s,-10,10\right),$$

$$\text{parametric}\left(\frac{1}{s^2+1},-\left(\frac{s}{s^2+1}\right),s,-10,10\right]$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-3,3],
yaxis=true,yaxis_type=solid,yrange=[-3,3],
proportional_axes=xy,
   nticks=600,
v1,
color=red,
h1);
```



practical -3
plot of the ellipse(2cos(t),sin(t)),t in [0,2pi]

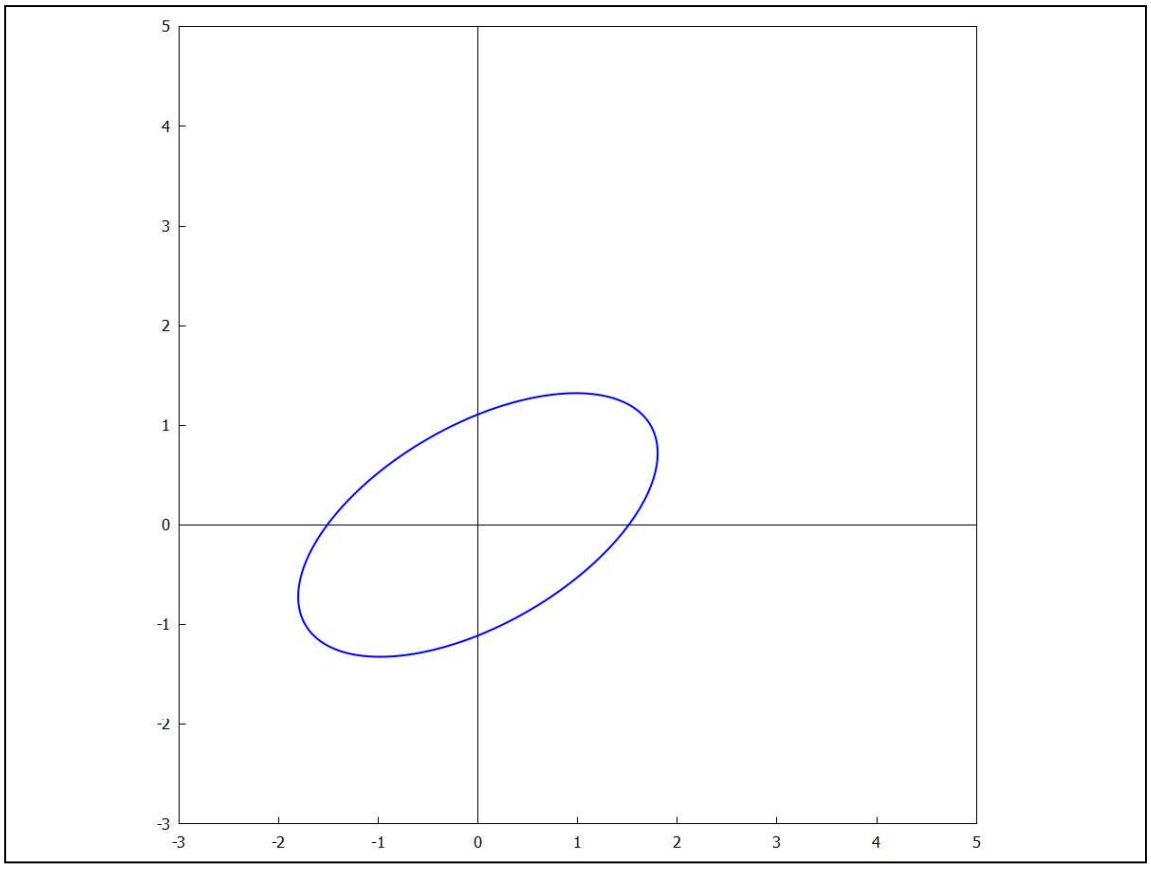
```
kill(all);
```

done

```
s(t):=2·cos(t)+%i·sin(t);
```

$$s(t):=2\cos(t)+\%i\sin(t)$$

```
wxdraw2d(
  xaxis=true,xaxis_type=solid,xrange=[-3,5],
  yaxis=true,yaxis_type=solid,yrange=[-3,5],
  proportional_axes=xy,
  nticks=200,
  line_width=2,
  parametric(realpart(r(t)),imagpart(r(t)), t,0, 2·%pi)
  );
```



rotate the ellipse about the origin by an angle of pi/6

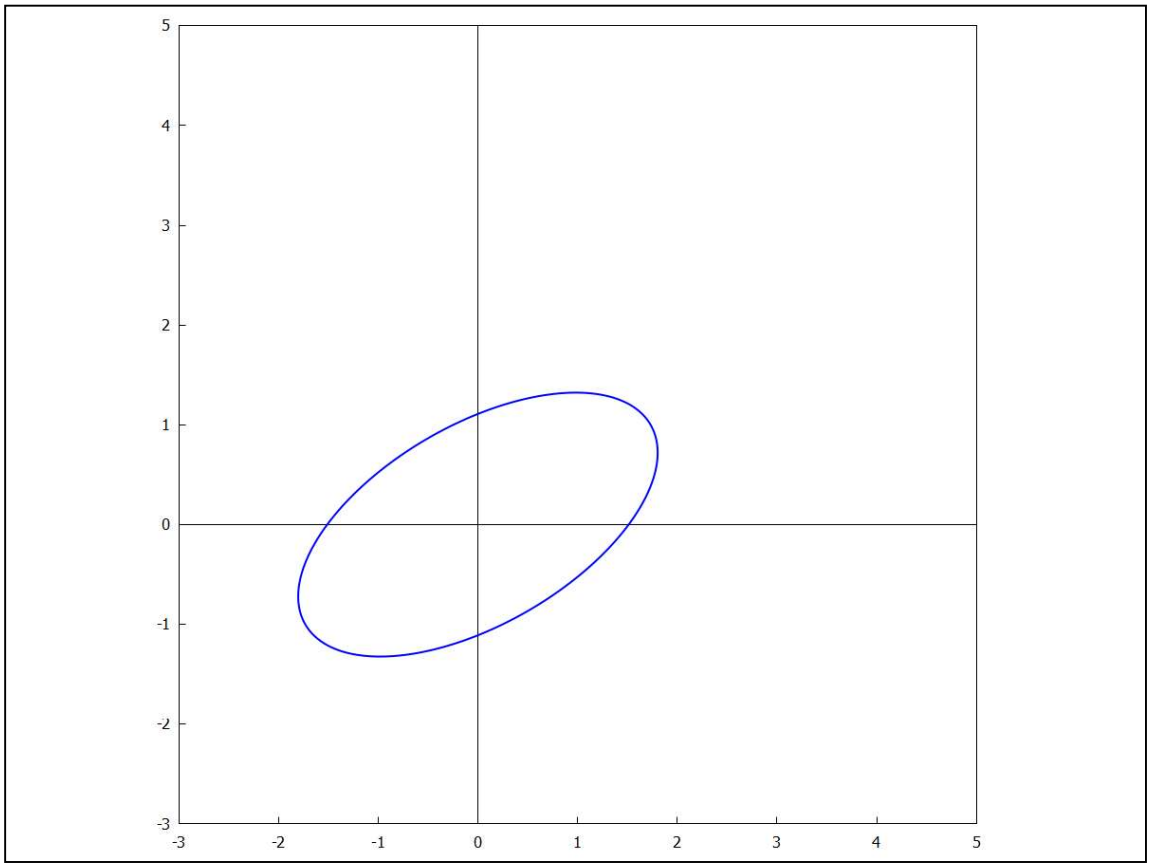
```
;theta:%pi/6;
```

$$\frac{\pi}{6}$$

```
r(t):=s(t)·exp(%i·theta);
```

$$r(t) := s(t)\,\exp(\%i\,theta)$$

```
wxdraw2d(
  xaxis=true,xaxis_type=solid,xrange=[-3,5],
  yaxis=true,yaxis_type=solid,yrange=[-3,5],
  proportional_axes=xy,
  nticks=200,
  line_width=2,
  parametric(realpart(r(t)),imagpart(r(t)), t,0, 2·%pi)
  );
```

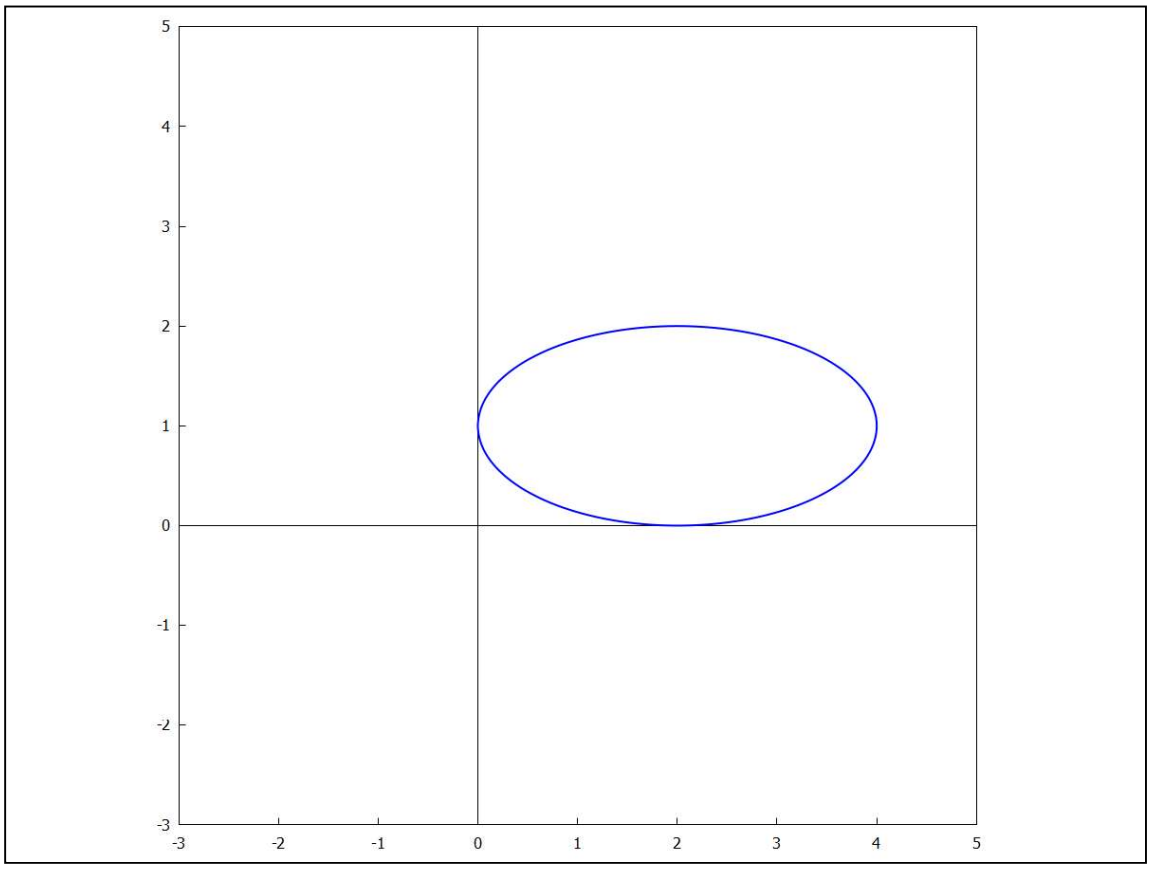


shift the centre of the ellipse (2cos(t),sin(t)),t in [0,2pi] to (2,1)

```
r(t):=s(t)+(2+%i);
```

$$r(t) := s(t)+(2+\%i)$$

```
wxdraw2d(
  xaxis=true,xaxis_type=solid,xrange=[-3,5],
  yaxis=true,yaxis_type=solid,yrange=[-3,5],
  proportional_axes=xy,
  nticks=200,
  line_width=2,
  parametric(realpart(r(t)),imagpart(r(t)), t,0, 2·%pi)
  );
```



rotate the ellipse by pi/6 and shift the centre of the ellipse to (2,1)

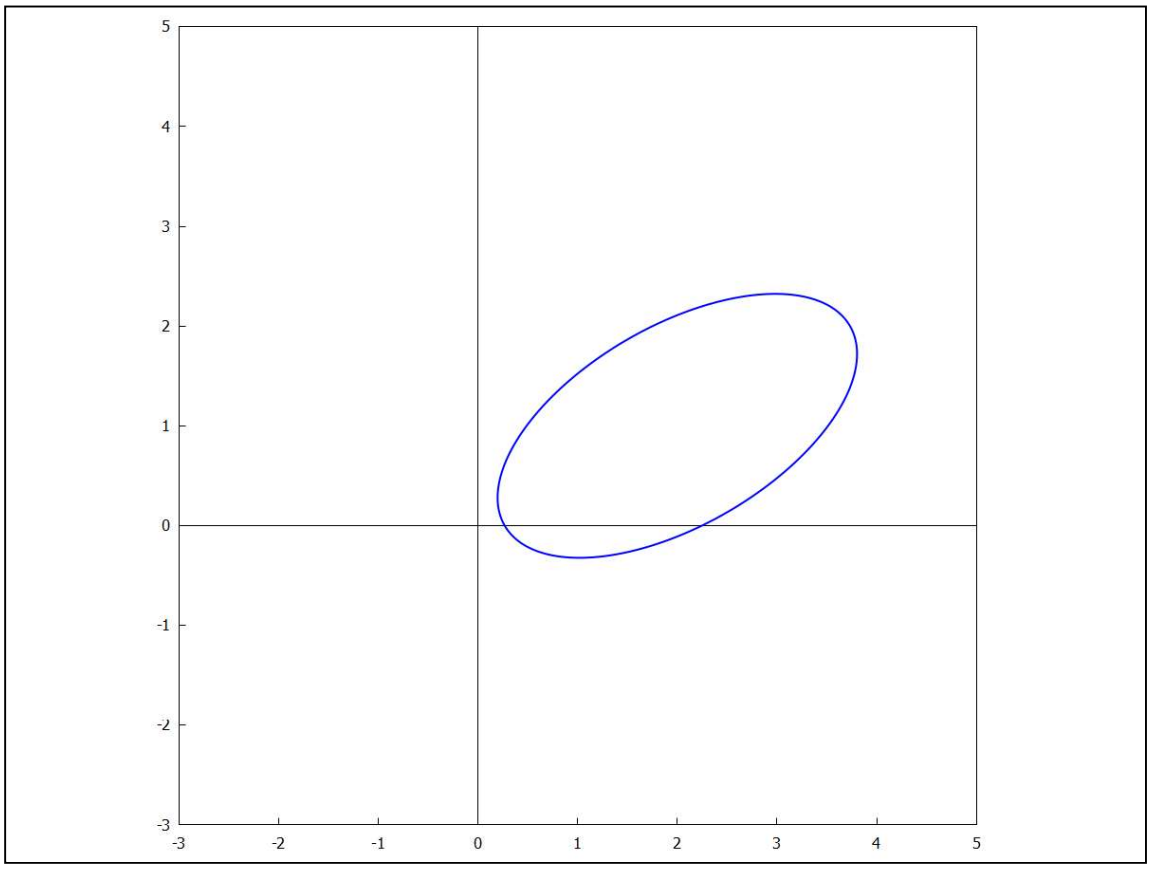
```
theta:%pi/6;
```

$$\frac{\pi}{6}$$

```
r(t):=s(t)·exp(%i·theta)+(2+%i);
```

$$r(t):=s(t)\exp(\%i\ theta)+(2+\%i)$$

```
wxdraw2d(
  xaxis=true,xaxis_type=solid,xrange=[-3,5],
  yaxis=true,yaxis_type=solid,yrange=[-3,5],
  proportional_axes=xy,
  nticks=200,
  line_width=2,
  parametric(realpart(r(t)),imagpart(r(t)), t,0, 2·%pi)
  );
```



PRACTICAL -8
find a parametrization of the polygonal path c=C1+C2+C3
FROM -1+i to 3-i,
where c1 is -1+i to -1
c2 is -1 to 1+i
c3 is 1+i to 3-i

```
kill(all);
```

done

```
z1(t):=(-1)+%i·(1-t);
```

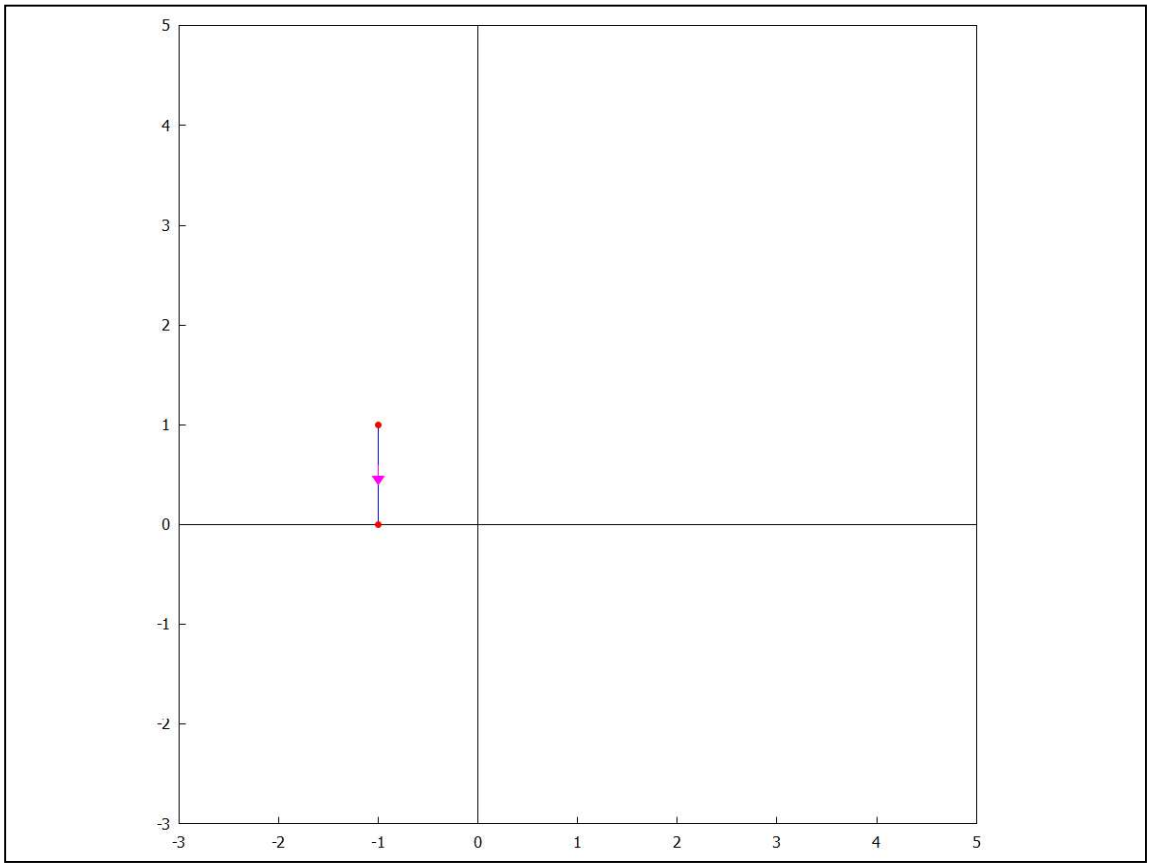
$$z1(t):=-1+\%i\,(1-t)$$

PRACTICAL -8
find a parametrization of the polygonal path c=C1+C2+C3
FROM -1+i to 3-i,
where c1 is -1+i to -1
c2 is -1 to 1+i
c3 is 1+i to 3-i

```
kill( all);
z1( t ):=-1+%i (1-t );
z1( t):=( -1)+%i·( 1-t);
```

done z1(t):=−1+%i(1−t) z1(t):=−1+%i (1−t)

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[ -3,5] ,
yaxis=true,yaxis_type=solid,yrange=[ -3,5] ,
proportional_axes=xy ,
parametric( realpart( z1( t)), imagpart( z1( t)),t,0,1),
head_length=0.2,
head_angle=20,
color =magenta,
vector ( [ -1,0.6] ,[ 0,-0.2] ),
color =red,
point_type=7,
point_size=1,
points( [ [ -1,1] ,[ -1,0] ] )
);
```



C2

```
z2( t):=( -1+2·t)+%i·t;
```

$$z2(t):=-1+2\,t+\%i\,t$$

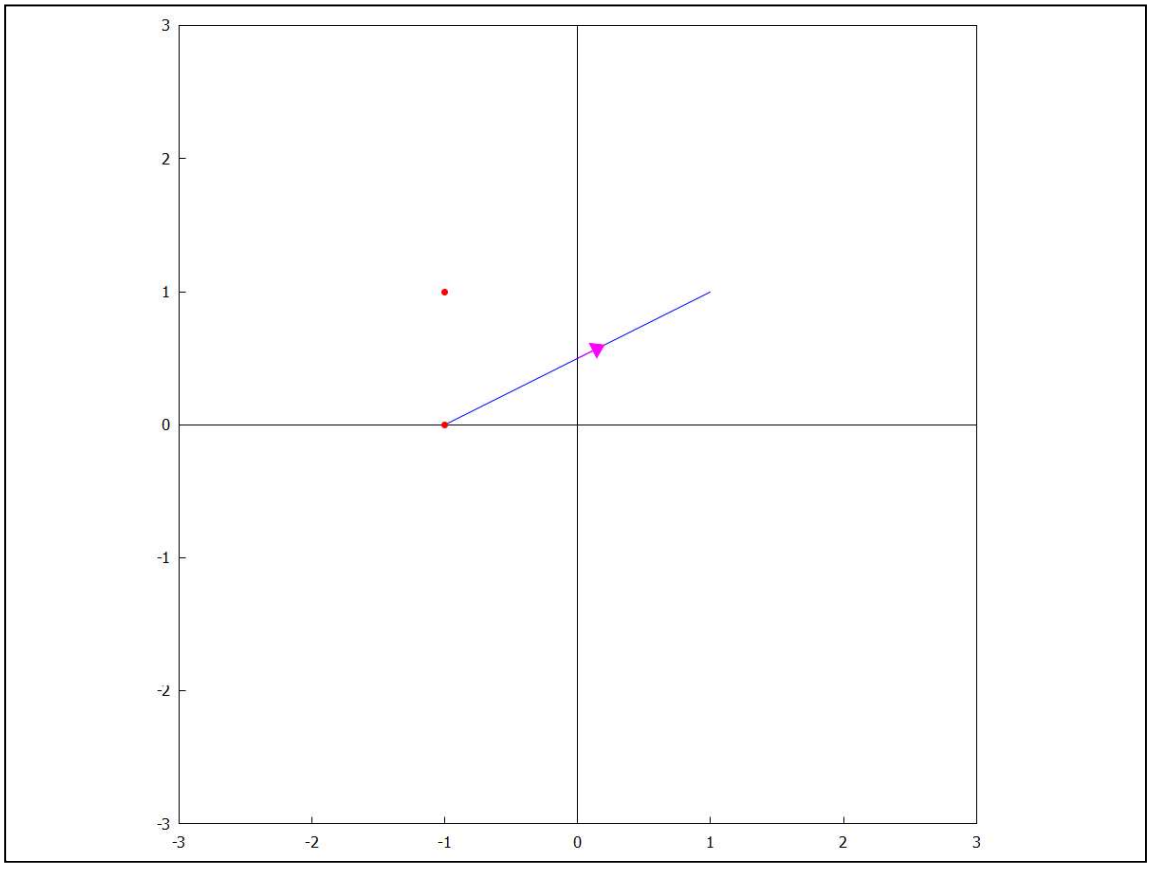
```
z2( 1/ 2);
```

$$\frac{\%i}{2}$$

```
z2(1/3);
```

$$\frac{\%i}{3}-\frac{1}{3}$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[ -3,3] ,
yaxis=true,yaxis_type=solid,yrange=[ -3,3] ,
proportional_axes=xy ,
parametric( realpart( z2( t)), imagpart( z2( t)),t,0,1),
head_length=0.2,
head_angle=20,
color =magenta,
vector ( [ 0,1/ 2] , [ 2/ 10, 1/ 10]),
color =red,
point_type=7,
point_size=1,
points( [ [ -1,1] ,[ -1,0] ] )
);
```



c3

```
z3(t):=(1+2·t)+%·(1-2·t);
```

$$z3(t):=1+2\,t+\%\,(1-2\,t)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[ -3,4] ,
yaxis=true,yaxis_type=solid,yrange=[ -3,4] ,
proportional_axes=xy ,
parametric( realpart( z3( t)), imagpart( z3( t)),t,0,1),
head_length=0.2,
head_angle=20,
color =magenta,
vector ( [ 2,0] , [ 2/ 10, -2/ 10]),
color =red,
point_type=7,
point_size=1,
points( [ [ 1,1] ,[ 3,-1] ] )
);
```

```
draw2d (parametric): non defined variable
 -- an error. To debug this try: debugmode(true);
```

c1+c2+c3

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[ -3,4] ,
yaxis=true,yaxis_type=solid,yrange=[ -3,4] ,
proportional_axes=xy ,
 parametric( realpart( z1( t)), imagpart( z1( t)),t,0,1),
 parametric( realpart( z2( t)), imagpart( z2( t)),t,0,1),
 parametric( realpart( z3( t)), imagpart( z3( t)),t,0,1)
head_length=0.2,
head_angle=20,
 vector([-1,0.8],[0,-0.4]),
 vector ( [ 0,1/ 2] , [ 2/ 10, 1/ 10]),
 vector([2,0], [2/10, -2/10]),
color =red,
point_type=7,
point_size=1,
points( [ [ -1,1] ,[ -1,0] ,[1,1],[3,-1]])
   );
```

```
incorrect syntax: head_length is not an infix operator
head_length=
       ^
incorrect syntax: Too many )'s
   )
  ^
```

Practical : 9

Plot the line segment 'L' joining the point A=0 to B= 2+%PI/4 i
and give an exact calculation of int(e^z)dz over L.

```
kill(all);
```

done

```
z(t):=t+%i·((%pi/8)·t);
```

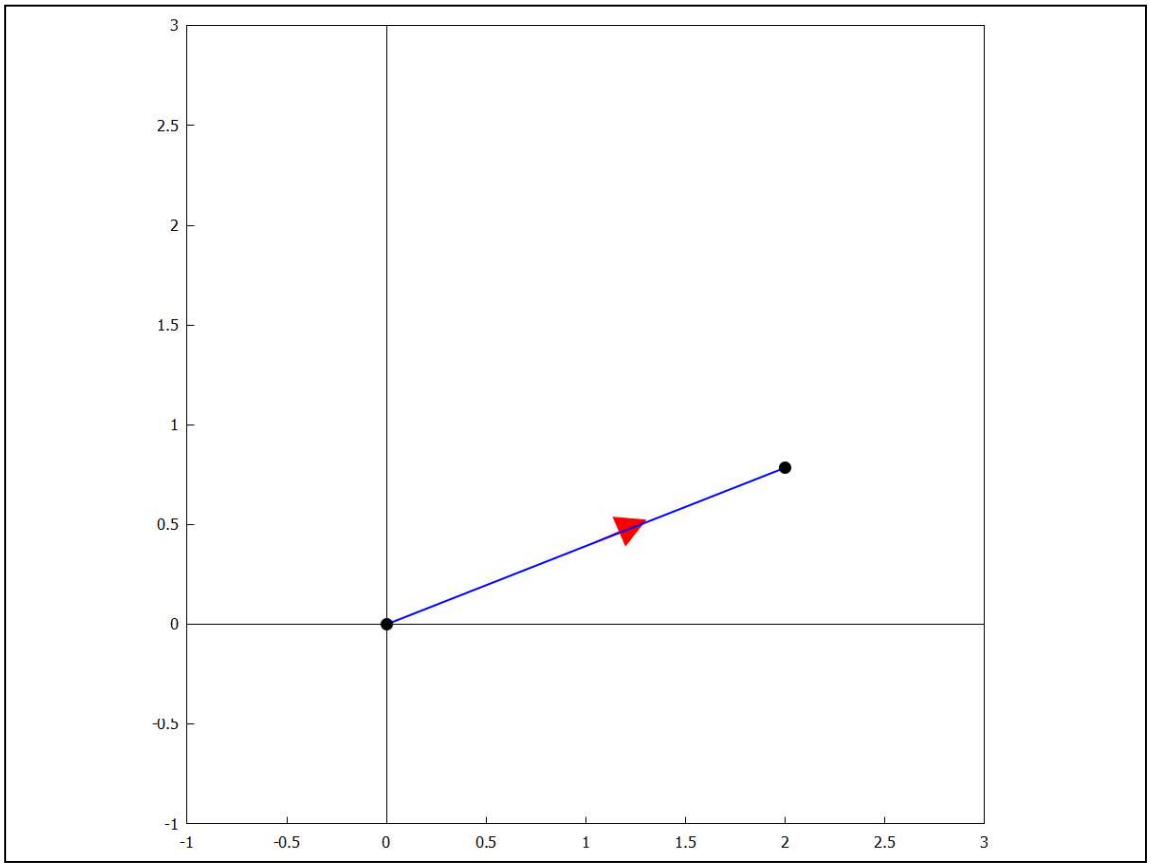
$$z(t):=t+\%i\left(\frac{\pi}{8}t\right)$$

```
wxdraw2d(
xaxis=true, xaxis_type=solid,xrange=[-1,3],
yaxis=true,yaxis_type=solid,yrange=[-1,3],
proportional_axes=xy,

head_length=0.5,
head_angle=10,
color=red,
vector([1, %pi/8],[0.3,0.13]),

color=blue,
line_width=2,
parametric(realpart(z(t)), imagpart(z(t)),t,0,2),

color=black,
point_type=7,
point_size=2,
points([[realpart(z(0)),imagpart(z(0))],[realpart(z(2)),imagpart(z(2))]]));
```



Evaluate int(e^z dz) over L

```
kill(all);
```

done

```
cIntegral(p,q,a,b):=block(
f(z):=exp(z),
g(t):=(p)+%i·(q),
rectform(integrate(rectform(f(g(t))·diff(g(t),t)),t,a,b)));
```

$$\text{cIntegral}(p,q,a,b):=\text{block}(f(z):=\exp(z),g(t):=p+\%i\,q,\text{rectform}\left(\int_a^b \text{rectform}\left(f(g(t))\left(\frac{d}{dt}g(t)\right)\right)dt\right))$$

```
cIntegral(t,(%pi/8)·t,0,2);
```

$$\frac{\%e^{2\,\%i}}{\sqrt{2}}+\frac{\%e^{2-\sqrt{2}}}{\sqrt{2}}$$

Practical : 10

Evaluate integral 1/(z-2) dz , where c is upper semicircle with radius 1 centered at z=2 oriented positive direction.

```
kill(all);
```

done

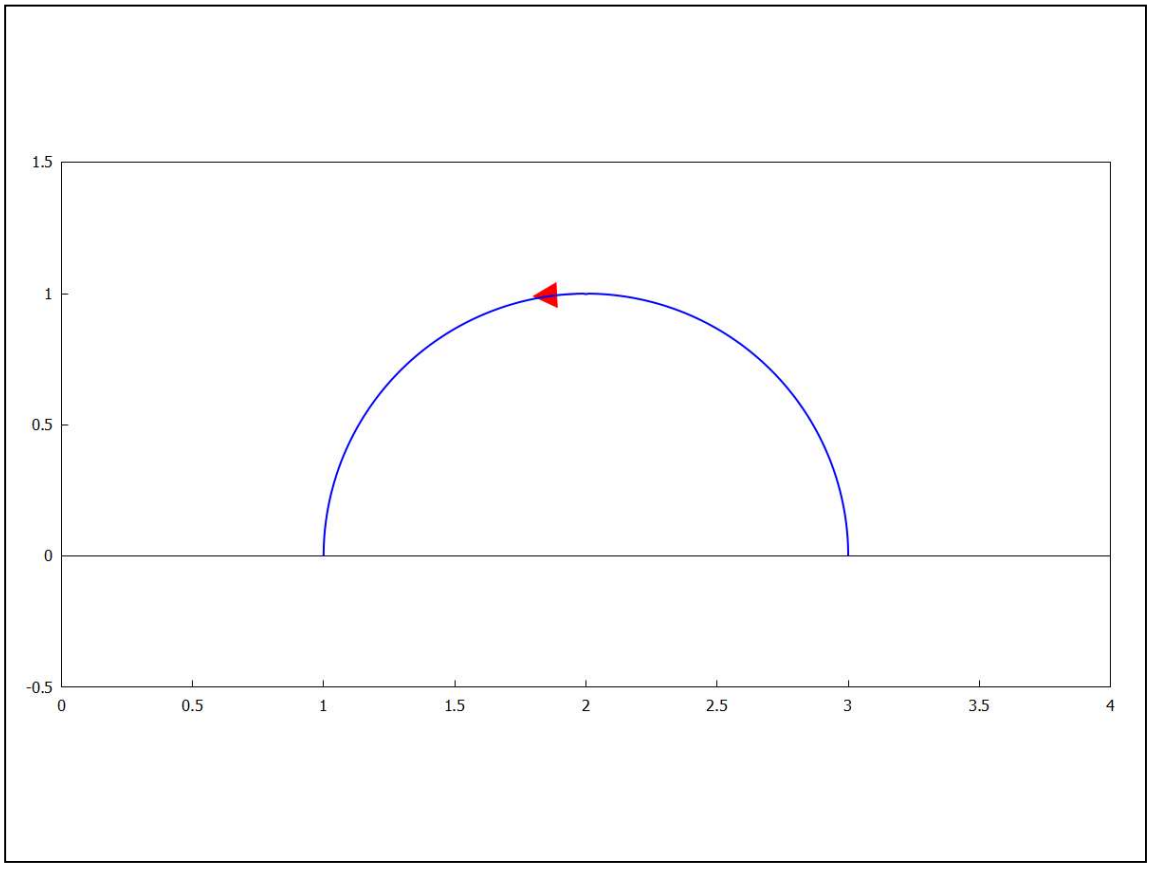
```
z(t):=(2+cos(t))+ %i·sin(t);
```

$$z(t):=2+\cos(t)+\%i\,\sin(t)$$

```
wxdraw2d(
xaxis=true, xaxis_type=solid,xrange=[0,4],
yaxis=true,yaxis_type=solid,yrange=[-1/2,3/2],
proportional_axes=xy,

head_length=0.3,
head_angle=10,
color=red,
vector([2,1],[-0.2 , -0.01]),

color=blue,
line_width=2,
   nticks=500,
parametric(realpart(z(t)), imagpart(z(t)),t,0,%pi))
;
```



```
cintegral(p,q,a,b):=block(
f(z):= 1/(z−2),
g(t):=p+%i·(q),
rectform(integrate(rectform(f(g(t))·diff(g(t),t)),t,a,b)));
```

$$\text{cintegral}(p,q,a,b):=\text{block}\left(\text{f}(z):=\frac{1}{z-2},\text{g}(t):=p+\%\text{i}\,q,\text{rectform}\left(\int_a^b \text{rectform}\left(\text{f}(\text{g}(t))\left(\frac{\text{d}}{\text{d}\,t}\,\text{g}(t)\right)\right)\text{d}t\right)\right)$$

```
cintegral(2+cos(t),sin(t),0,%pi);
```

$$\%\text{i}\,\pi$$

Practical : 11

Show that int(z dz) over c1 = int(z dz) over c2 = 4+2i

```
kill(all);
```

done

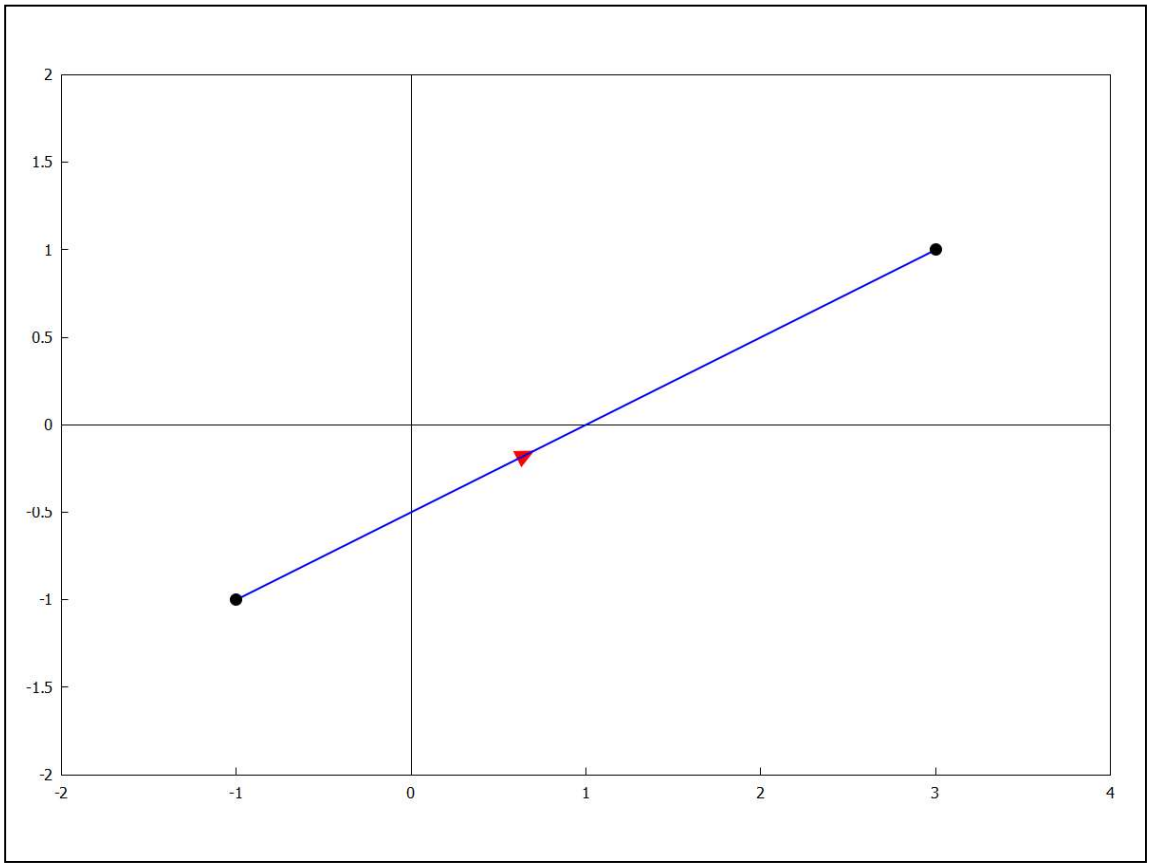
```
z(t):=(t)+%i·(1+((1/2)·(t−3)));
```

$$\text{z}(t):=t+\%\text{i}\left(1+\frac{1}{2}\,(t-3)\right)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,4],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,

head_length=0.3,
head_angle=10,
color=red,
vector([1/2,−1/4],[1/5 , 1/10]),

nticks=500,
color=blue,
line_width=2,
parametric(realpart(z(t)),imagpart(z(t)),t,−1,3),
color=black,
point_type=7,
point_size=2,
points([[realpart(z(3)),imagpart(z(3))],[realpart(z(−1)),imagpart(z(−1))]]));
```




```
kill(all);
```

done

Along C_1

```
cintegral(p,q,a,b):=block(
f(z):=z,
z(t):=p+%i·(q),
rectform(integrate(rectform(f(z(t))·diff(z(t),t)),t,a,b)));
```

$$\text{cintegral}(p,q,a,b):=\text{block}(\text{f}(z):=z,\text{z}(t):=p+\%\text{i}\,q,\text{rectform}\left(\int_a^b \text{rectform}\left(\text{f}(\text{z}(t))\left(\frac{\text{d}}{\text{d}\,t}\,\text{z}(t)\right)\right)\text{d}t\right))$$

```
cintegral(t, 1+((1/2)·(t−3)),−1,3);
```

$$2\,\%\text{i}+4$$

Along C_2

```
kill(all);
```

done

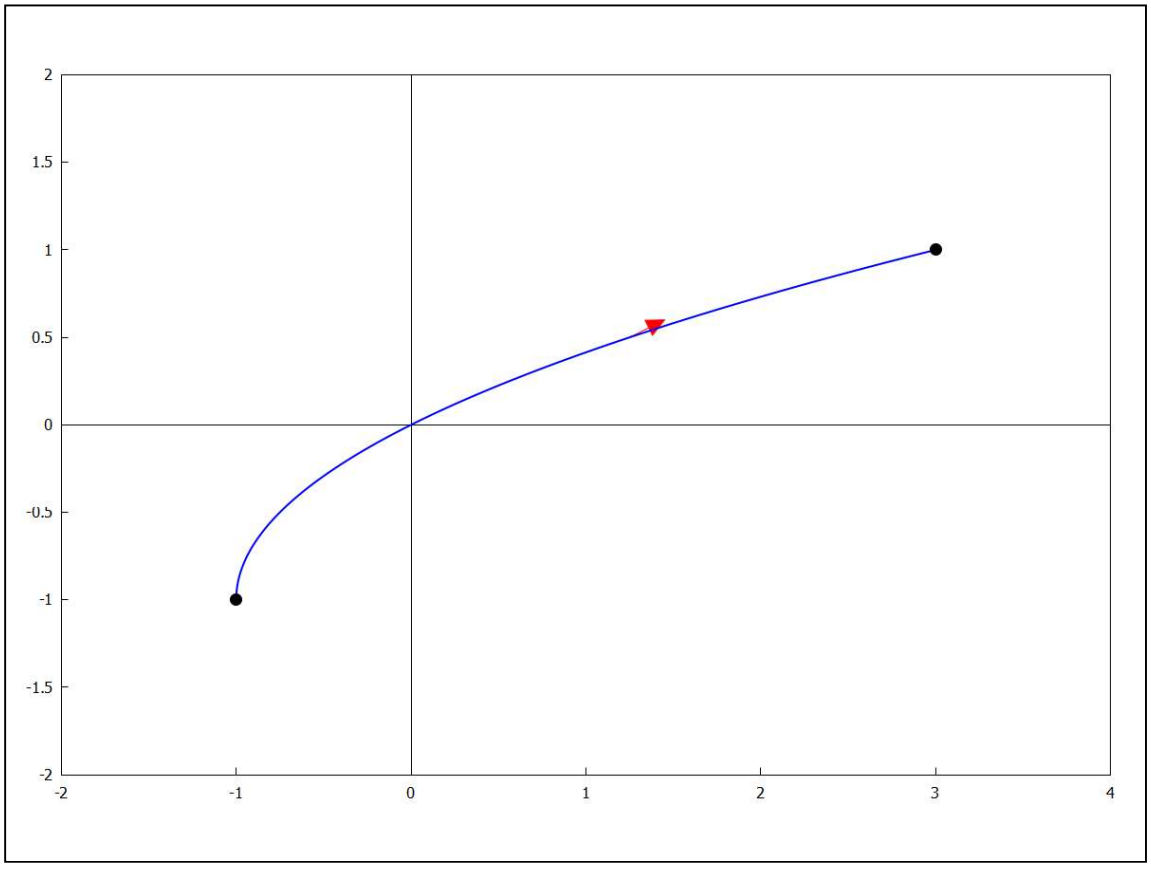
```
z(t):=(t^2+2·t)+%i·(t);
```

$$z(t) := t^2 + 2\,t + \%i\,t$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-2,4],
yaxis=true,yaxis_type=solid,yrange=[-2,2],
proportional_axes=xy,

head_length=0.3,
head_angle=10,
color=red,
vector([5/4,1/2],[1/5 , 1/10]),

nticks=500,
color=blue,
line_width=2,
parametric(realpart(z(t)),imagpart(z(t)),t,-1,1),
color=black,
point_type=7,
point_size=2,
points([[realpart(z(1)),imagpart(z(1))],[realpart(z(-1)),imagpart(z(-1))]]));
```



```
cintegral(p,q,a,b):=block(
f(z):=z,
z(t):=p+%i·(q),
rectform(integrate(rectform(f(z(t))·diff(z(t),t)),t,a,b)));
```

$$\text{cintegral}(p,q,a,b):=\text{block}(\text{f}(z):=z,\text{z}(t):=p+\%\text{i}\,q,\text{rectform}\left(\int_a^b \text{rectform}\left(\text{f}(\text{z}(t))\left(\frac{\text{d}}{\text{d}\,t}\,\text{z}(t)\right)\right)\text{d}\,t\right))$$

```
cintegral(t,(t^2+2·t),−1,1);
```

$$2\,\%\text{i}-4$$

Practical : 12 skip

Practical: 13

Find and plot three different Laurent series representations for the function:

f(z)=3/2+z-z^2 , involving powers of z.

```
kill(all);
```

done

```
f(z):=3/(2+z−z^2);
```

$$\text{f}(z):=\frac{3}{2+z-z^2}$$

```
g(z):=1/(1+z);
```

$$\text{g}(z):=\frac{1}{1+z}$$

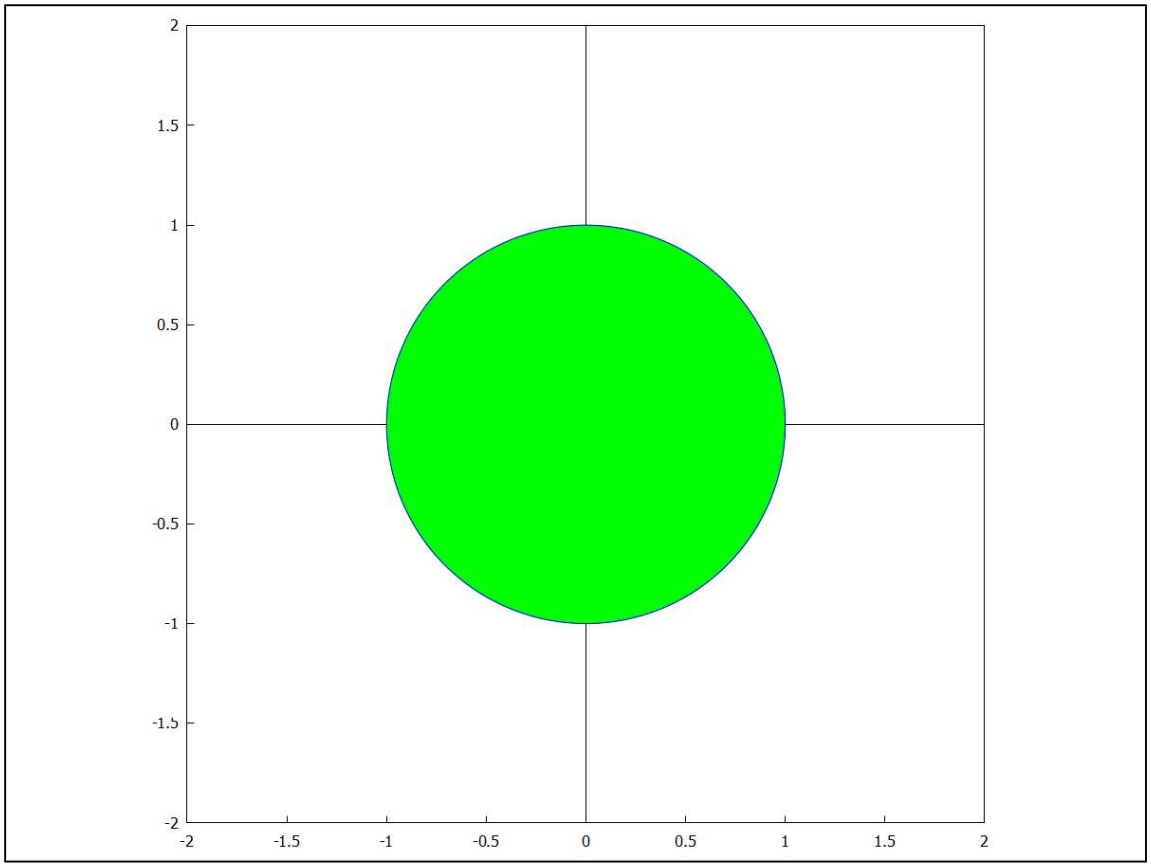
```
h(z):=(1/2)·(1/(1−(z/2)));
```

$$h(z):=\frac{1}{2}\,\frac{1}{1-\frac{z}{2}}$$

|z| <1

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-2,2],
yaxis=true,yaxis_type=solid,yrange=[-2,2],
proportional_axes=xy,

nticks=200,
fill_color=green,
ellipse(0,0,1,1,0,360));
```




```
f(z):=3/(2+z-z^2);
```

$$f(z):=\frac{3}{2+z-z^2}$$

```
g(z):=1/(1+z);
```

$$g(z):=\frac{1}{1+z}$$

```
h(z):=(1/2)·(1/(1-(z/2)));
```

$$h(z):=\frac{1}{2}\,\frac{1}{1-\frac{z}{2}}$$

```
taylor(g(z),z,0,4);
```

$$1-z+z^2-z^3+z^4+\ldots$$

```
taylor(h(z),z,0,4);
```

$$\frac{1}{2}+\frac{z}{4}+\frac{z^2}{8}+\frac{z^3}{16}+\frac{z^4}{32}+\ldots$$

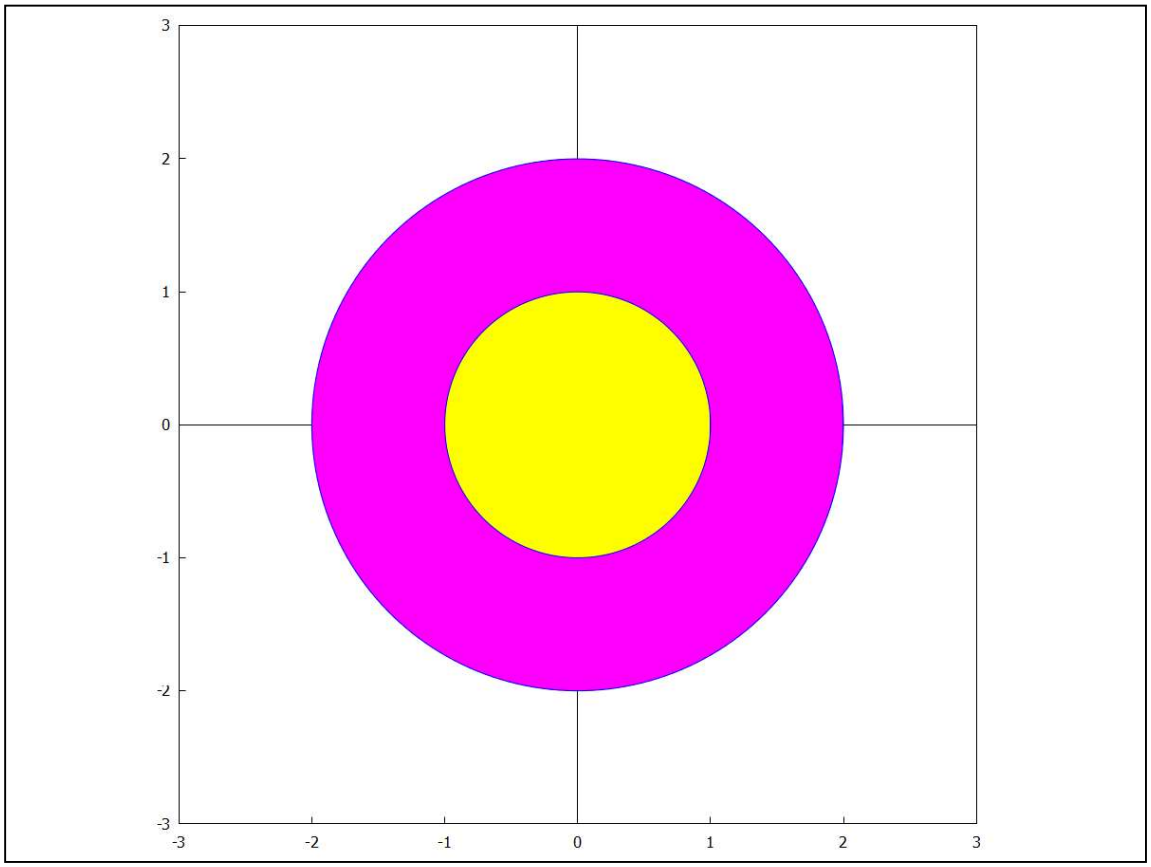
```
taylor(g(z),z,0,4)+taylor(h(z),z,0,4);
```

$$\frac{3}{2}-\frac{3\,z}{4}+\frac{9\,z^2}{8}-\frac{15\,z^3}{16}+\frac{33\,z^4}{32}+\ldots$$

$1 < |z| < 2$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-3,3],
yaxis=true,yaxis_type=solid,yrange=[-3,3],
proportional_axes=xy,

nticks=200,
fill_color=magenta,
ellipse(0,0,2,2,0,360),
fill_color=yellow,
ellipse(0,0,1,1,0,360));
```




```
f(z):=3/(2+z-z^2);
```

$$f(z):=\frac{3}{2+z-z^2}$$

```
g(z):=1/(1+z);
```

$$g(z):=\frac{1}{1+z}$$

```
h(z):=(1/2)·(1/(1-(z/2)));
```

$$h(z):=\frac{1}{2}\,\frac{1}{1-\frac{z}{2}}$$

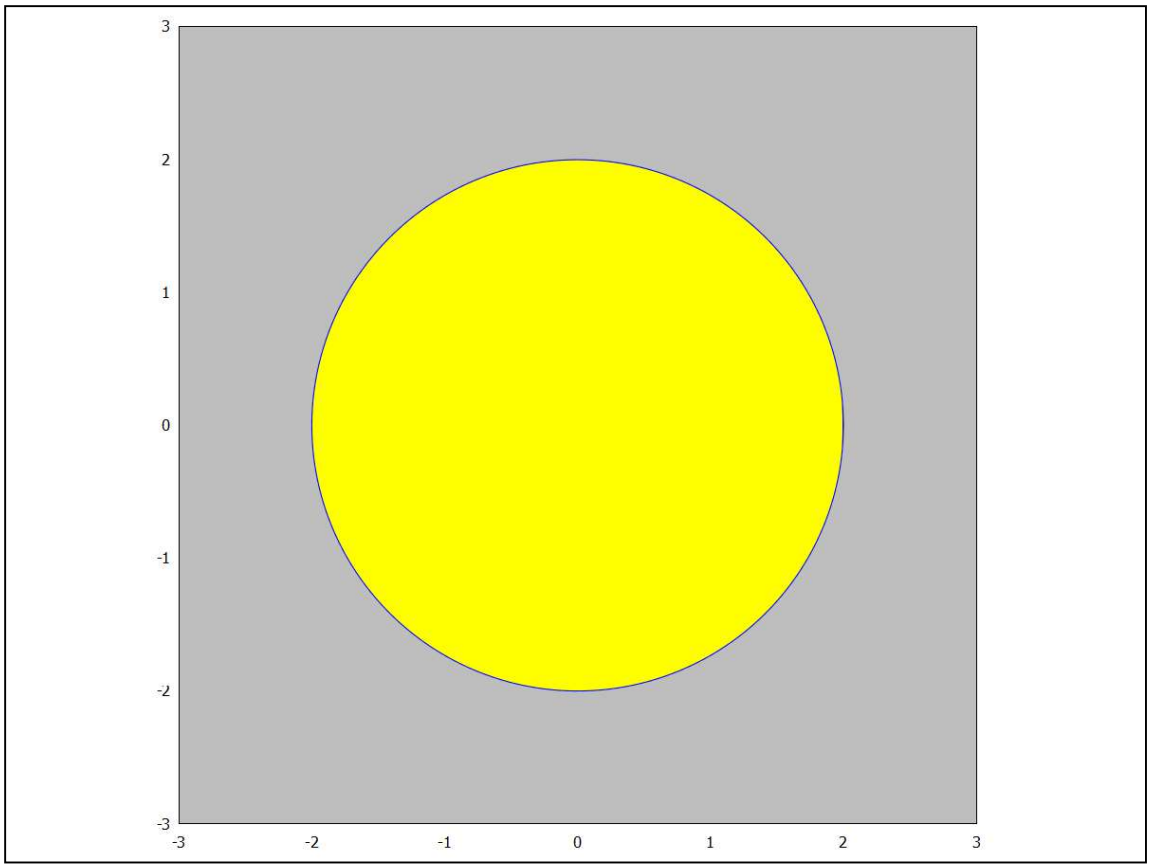
```
taylor(g(z),[z,0,4,'asymp])+taylor(h(z),z,0,4);
```

$$\frac{z^4}{32}+\frac{z^3}{16}+\frac{z^2}{8}+\frac{z}{4}+\frac{1}{2}+\frac{1}{z}-\frac{1}{z^2}+\frac{1}{z^3}-\frac{1}{z^4}+\ldots$$

|z| >2

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-3,3],
yaxis=true,yaxis_type=solid,yrange=[-3,3],
proportional_axes=xy,

nticks=200,
fill_color=gray,
ellipse(0,0,6,6,0,360),
fill_color=yellow,
ellipse(0,0,2,2,0,360));
```




```
f(z):=3/(2+z-z^2);
```

$$f(z):=\frac{3}{2+z-z^2}$$

```
g(z):=1/(1+z);
```

$$g(z):=\frac{1}{1+z}$$

```
h(z):=(1/2)·(1/(1-(z/2)));
```

$$h(z):=\frac{1}{2}\,\frac{1}{1-\frac{z}{2}}$$

```
taylor(g(z),[z,0,4,'asymp])+taylor(h(z),[z,0,4,'asymp]);
```

$$-\left(\frac{3}{z^2}\right)-\frac{3}{z^3}-\frac{9}{z^4}+\ldots$$

Practical: 14

Locate the poles of f(z)=1/5z^4+26z^2+5 and specify their order.

```
kill(all);
```

done

```
load(coma);
```

coma v.2.1 (Wilhelm Haager, 2019−05−21)
C:/maxima−5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac

```
zeros(s^2+5);
```

$$[2.2361\ \%i, -(2.2361\ \%i)]$$

```
zeros(5·s^2+1);
```

$$[0.44721\ \%i, -(0.44721\ \%i)]$$

```
zeros(5·s^4+26·s^2+5);
```

$$[0.44721\ \%i, -(0.44721\ \%i), -(2.2361\ \%i), 2.2361\ \%i]$$

```
zeros(1/(5·s^4+26·s^2+5));
```

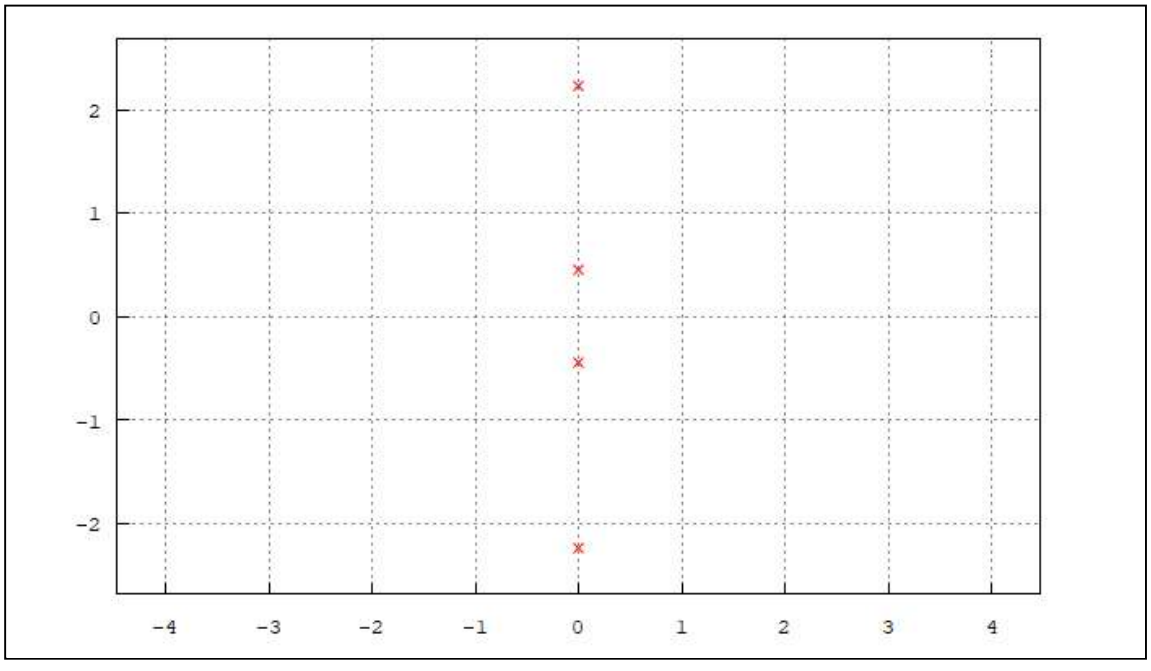
[]

```
poles(1/(5·s^4+26·s^2+5));
```

$$[0.44721\ \%i, -(0.44721\ \%i), -(2.2361\ \%i), 2.2361\ \%i]$$

% poles of order 1 , simple pole

```
poles_and_zeros(1/(5·s^4+26·s^2+5));
```



```
float(sqrt(5));
```
2.2361
```
float(1/(sqrt(5)));
```
0.44721

Practical : 15

Locate the zeros and poles of g(z)=%pi*cot(%pi*z)/z^2 and determine their order. Also justify that Res(g,0)=- %pi^3/3.

```
kill(all);
```
done
```
load(coma);
```
coma v.2.1 (Wilhelm Haager, 2019-05-21)
C:/maxima-5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac
```
g(s):= %pi·cot(%pi·s)/s^2;
```
$$g(s):=\frac{\pi\cot(\pi s)}{s^2}$$
```
zeros(g(s));
```
[ ]
```
poles(g(s));
```
[ ]
```
poles_and_zeros(g(s));
```
draw: illegal values in range specification
#0: poles_and_zeros(f=(%pi*cot(%pi*s))/s^2,opts=[]) (coma.mac line 442)
 -- an error. To debug this try: debugmode(true);
```
residue(g(s),s,0);
```
$$-\left(\frac{\pi^2}{3}\right)$$

Example : Find Res[g,0] if g(z)=3/2z+z^2-z^3

```
kill(all);
```
done
```
load(coma);
```
coma v.2.1 (Wilhelm Haager, 2019-05-21)
C:/maxima-5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac
```
g(s):= 3/(2·s+s^2-s^3);
```

$$g(s):=\frac{3}{2s+s^2-s^3}$$

```
zeros(2·s+s^2−s^3);
```

$$[0,-1.0,2.0]$$

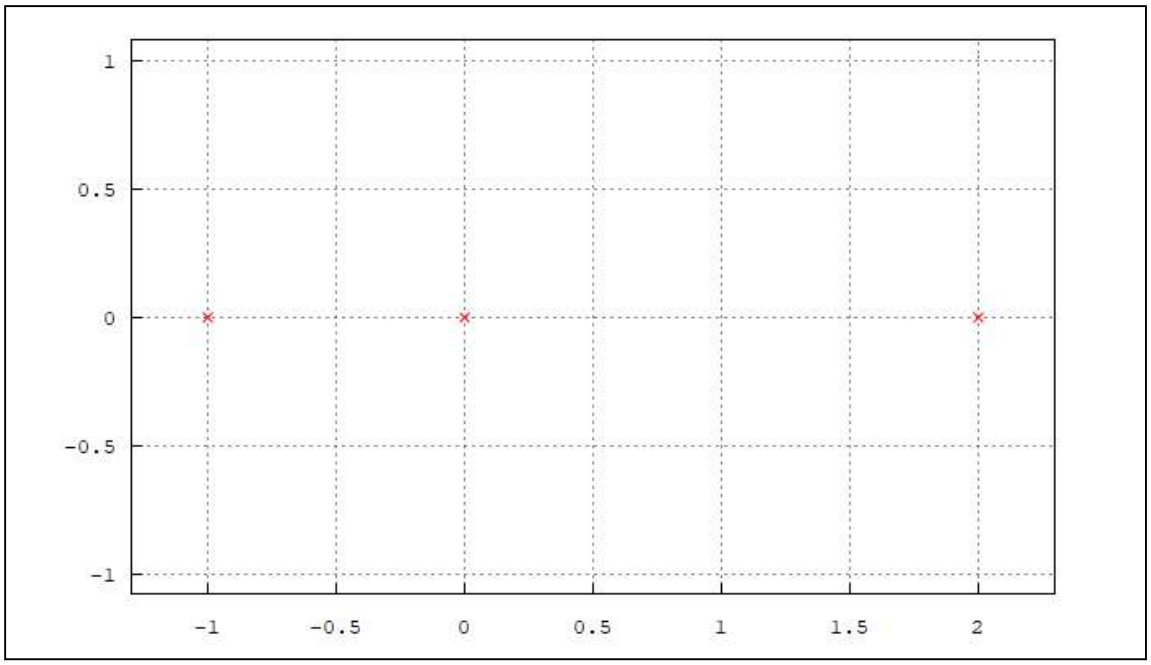
```
zeros(1/(2·s+s^2−s^3));
```

$$[\,]$$

```
poles(1/(2·s+s^2−s^3));
```

$$[0,-1.0,2.0]$$

```
poles_and_zeros(1/(2·s+s^2−s^3));
```




```
residue(g(s),s,0);
```

$$\frac{3}{2}$$

```
residue(g(s),s,−1);
```

$$-1$$

Practical : 16

Evaluate %int exp^(2/z) dz . where c_1(0) denotes the circle {z:|z|=1} with positive orientation. similarly evaluate %int 1/(z^4+z^3-2z^2) dz.

```
kill(all);
```

done

```
load(coma);
```

coma v.2.1 (Wilhelm Haager, 2019-05-21)
C:/maxima-5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac

```
f(s):=exp(2/s);
```

$$f(s):=\exp\left(\frac{2}{s}\right)$$

```
residue(f(s),s,0);
```

0

```
taylor(f(s),[s,0,3, 'asymp]);
```

$$1+\frac{2}{s}+\frac{2}{s^2}+\frac{4}{3\,s^3}+\ldots$$

```
Int: 2·%pi· %i ·2;
```

4 %i π

Now we have to find another integration

```
kill(all);
```

done

```
load(coma);
```

coma v.2.1 (Wilhelm Haager, 2019-05-21)
C:/maxima-5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac

```
f(s):=1/(s^4+s^3 +(- 2)·s^2);
```

$$f(s):=\frac{1}{s^4+s^3+-2\,s^2}$$

```
zeros(s^2);
```

[0,0]

```
zeros(s^2+s-2);
```

[1.0,-2.0]

```
zeros(s^4+s^3-2·s^2);
```

[0,0,1.0,-2.0]

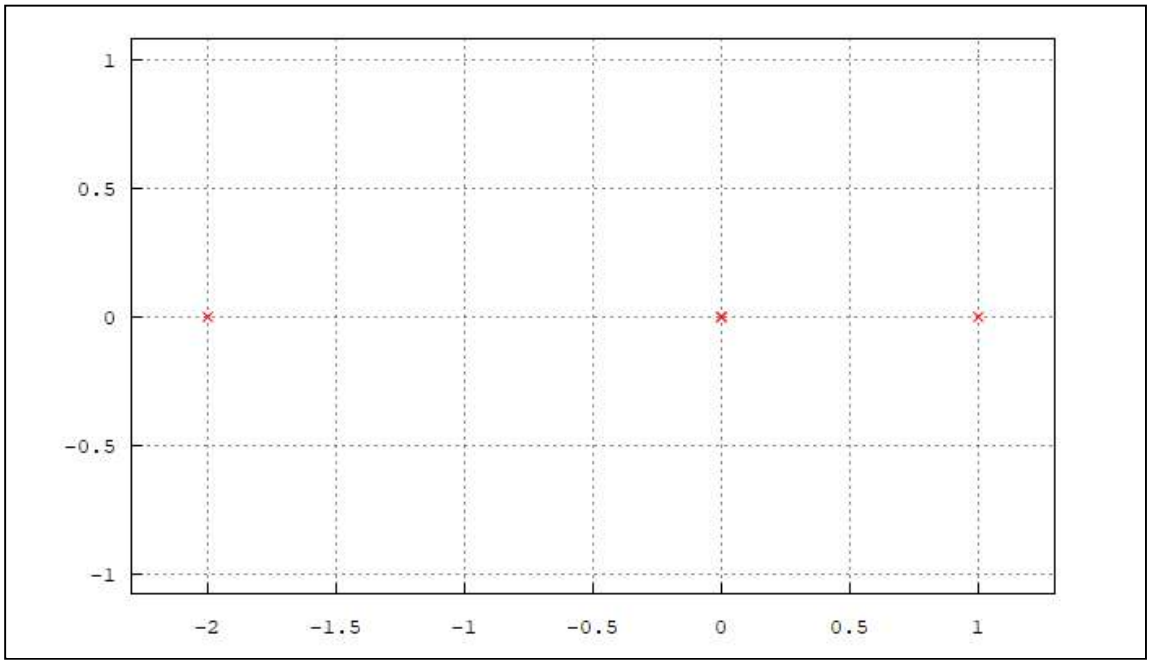
```
zeros(1/(s^4+s^3-2·s^2));
```

[]

```
poles(f(s));
```

[0,0,1.0,-2.0]

```
poles_and_zeros(f(s));
```




```
a:residue(f(s),s,0);
```

$$-\left(\frac{1}{4}\right)$$

```
b:residue(f(s),s,1);
```

$$\frac{1}{3}$$

```
c:residue(f(s),s,-2);
```

$$-\left(\frac{1}{12}\right)$$

```
Int: 2·%pi ·%i ·(a+b+c);
```

$$0$$